

25/02/2021
Evening



Aakash

Medical | IIT-JEE | Foundations

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Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2021 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS :

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each part has two sections.
 - (i) Section-I : This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) Section-II : This section contains 10 questions. In Section-II, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

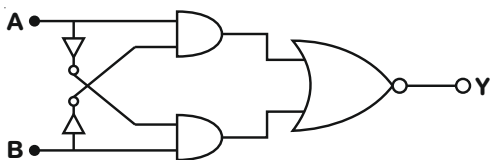
PART-A : PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The truth table for the following logic circuit is



(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(3)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Answer (3)

Sol. Truth table

A	B	$C = A \cdot \bar{B}$	$D = \bar{A} \cdot B$	$C + D$	$\overline{C + D} = Y$
0	0	0	0	0	1
0	1	0	1	1	0
1	0	1	0	1	0
1	1	0	0	0	1

2. Match List I with List II.

List I

(a) Rectifier

(b) Stabilizer

List II

(i) Used either for stepping up or stepping down the a.c. voltage

(ii) Used to convert a.c. voltage into d.c. voltage

(c) Transformer

(d) Filter

(iii) Used to remove any ripple in the rectified output voltage

(iv) Used for constant output voltage even when the input voltage or load current change

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(2) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(3) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(4) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Answer (2)

Sol. Correct match are

a → (ii)

b → (iv)

c → (i)

d → (iii)

3. A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is

(1) 35 m

(2) 45 m

(3) 25 m

(4) 50 m

Answer (2)

Sol. $5 = \frac{1}{2} \times 10 \times t_1^2 \Rightarrow t_1 = 1 \text{ s}$

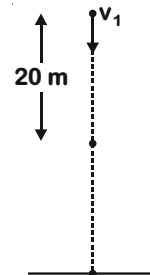
$\therefore v_1 = gt_1 = 10 \text{ m/s}$

Time of collision,

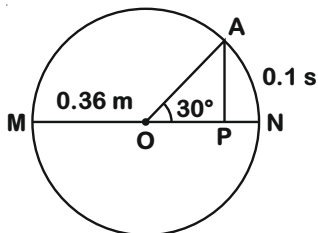
$\Delta t = \frac{20}{10} = 2 \text{ s}$

\therefore Total time of fall = 3 s

$\therefore H = \frac{1}{2} \times (10) \times 3^2 = 45 \text{ m}$



4. The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1 s. The perpendicular projection 'P' from 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be



- (1) 9.87 N (2) 50 N
(3) 100 N (4) 0.49 N

Answer (1)

Sol. $v = \frac{\left(\frac{\pi}{6}\right) \times (0.36)}{0.1} \text{ m/s}$

$= \pi \times 0.6 \text{ m/s}$

$\therefore a_M = \frac{v^2}{r} = \frac{(\pi \times 0.6)^2}{0.36} = \pi^2 = 9.87 \text{ N/kg}$

5. An LCR circuit contains resistance of 110Ω and a supply of 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45° . If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be

- (1) 1.5 A (2) 1 A
(3) 2 A (4) 2.5 A

Answer (3)

Sol. LCR circuit is in resonance.

$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{220}{110} = 2 \text{ A}$

6. Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter $0.1 \mu\text{m}$. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such that

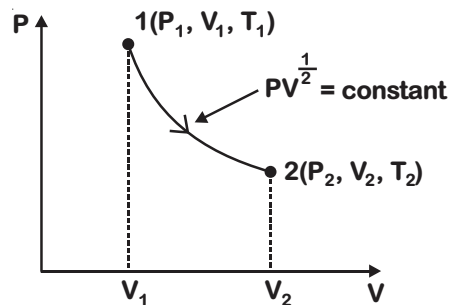
- (1) Its size decreases, and intensity decreases
(2) Its size increases, and intensity increases
(3) Its size increases, but intensity decreases
(4) Its size decreases, but intensity increases

Answer (4)

Sol. $\therefore r_{\text{dark}} = \frac{1.22 \lambda D}{b}$, where b is opening diameter.

When opening size is increased, the diffraction size decreases but intensity increases.

7. Thermodynamic process is shown below on a P-V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature T_2/T_1 is



- (1) $\sqrt{2}$ (2) 2
(3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{2}}$

Answer (1)

Sol. $PV^{1/2} = \text{constant}$

$\frac{T}{V} \times V^{1/2} = \text{constant}$

$\Rightarrow T \propto \sqrt{V}$

$\therefore \frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{2}{1}}$

$\Rightarrow \frac{T_2}{T_1} = \sqrt{2}$

8. An electron of mass m_e and a proton of mass $m_p = 1836m_e$ are moving with the same speed. The ratio of their de Broglie wavelength

$\frac{\lambda_{\text{electron}}}{\lambda_{\text{proton}}}$ will be

- (1) $\frac{1}{1836}$ (2) 918
(3) 1 (4) 1836

Answer (4)

Sol. $\lambda_e = \frac{h}{m_e \times v}$

$\lambda_p = \frac{h}{m_p \times v}$

$\Rightarrow \frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e} = 1836$

9. The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from $n = 2$ to $n = 1$ state is

- (1) 490.7 nm (2) 121.8 nm
(3) 913.3 nm (4) 194.8 nm

Answer (2)

Sol. $\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$
 $= R \times \left(\frac{3}{4} \right)$
 $\Rightarrow \lambda = \frac{4}{3R} = \frac{4}{3 \times 1.09 \times 10^7}$
 $= 121.8 \text{ nm}$

10. For extrinsic semiconductors; when doping level is increased;

- (1) Fermi-level of p-type semiconductors will go downward and Fermi-level of n-type semiconductor will go upward.
 (2) Fermi-level of p and n-type semiconductors will not be affected.
 (3) Fermi-level of both p-type and n-type semiconductors will go upward for $T > T_F$ K and downward for $T < T_F$ K, where T_F is Fermi temperature.
 (4) Fermi-level of p-type semiconductor will go upward and Fermi-level of n-type semiconductors will go downward.

Answer (1)

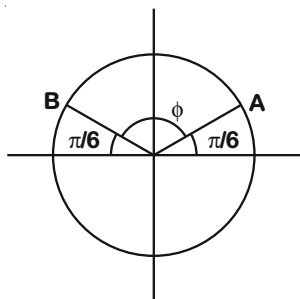
Sol. Fermi-level of p type semiconductor goes down whereas it goes up for n-type semiconductor.

11. $Y = A \sin(\omega t + \phi_0)$ is the time-displacement equation of a SHM. At $t = 0$ the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be:

- (1) $\frac{\pi}{6}$ (2) $\frac{2\pi}{3}$
(3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{3}$

Answer (3)

Sol. Draw phasor :



At B particle is going toward mean position \Rightarrow negative x-direction

$$\phi = \frac{5\pi}{6}$$

12. The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. The new wavelength is :

- (1) 329 nm (2) 400 nm
(3) 382 nm (4) 309 nm

Answer (3)

Sol. $\frac{hc}{\lambda} = \phi + eV$
 $\frac{hc}{\lambda_1} - eV_1 = \frac{hc}{\lambda_2} - eV_2$
 $\frac{1240}{491} - 0.71 = \frac{1240}{\lambda_2} - 1.43$
 $\frac{1240}{\lambda_2} = \frac{1240}{491} + 0.72$
 $\lambda_2 = 382 \text{ nm}$

13. If a message signal of frequency ' f_m ' is amplitude modulated with a carrier signal of frequency ' f_c ' and radiated through an antenna, the wavelength of the corresponding signal in air is:

- (1) $\frac{c}{f_c}$ (2) $\frac{c}{f_m}$
(3) $\frac{c}{f_c + f_m}$ (4) $\frac{c}{f_c - f_m}$

Answer (1)

Sol. In amplitude modulated wave, wave is transmitted at frequency of carrier wave

$$\lambda = \frac{c}{f_c}$$

14. Given below are two statements :

Statement I : In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement II : In a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.

In the light of the above statements, choose the correct answer from the options given below:

- (1) Both Statement I and Statement II are false.
- (2) Statement I is false but Statement II is true.
- (3) Statement I is true but Statement II is false.
- (4) Both Statement I and Statement II are true.

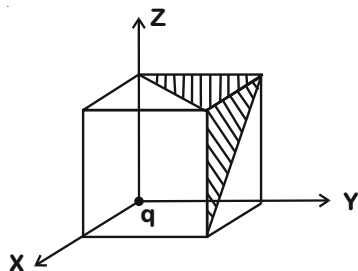
Answer (3)

Sol. $K_R = kT$

$$K_T = \frac{3}{2}kT$$

$$K_T \text{ for each molecule } \frac{3}{4}kT$$

15. A charge 'q' is placed at one corner of a cube as shown in figure. The flux of electrostatic field \vec{E} through the shaded area is:



- | | |
|------------------------------|------------------------------|
| (1) $\frac{q}{4\epsilon_0}$ | (2) $\frac{q}{24\epsilon_0}$ |
| (3) $\frac{q}{48\epsilon_0}$ | (4) $\frac{q}{8\epsilon_0}$ |

Answer (2)

Sol. Complete the cube with double the side with charge at centre

$$\text{flux, } \phi = \frac{1}{3} \times \frac{q}{8\epsilon_0} = \frac{q}{24\epsilon_0}$$

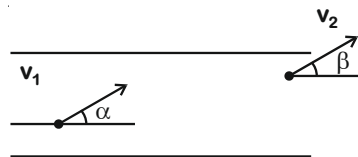
16. An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle ' α ' with the plates. It leaves the plates at angle ' β ' with kinetic energy K_2 . Then the ratio of kinetic energies $K_1 : K_2$ will be :

- (1) $\frac{\cos \beta}{\sin \alpha}$
- (2) $\frac{\cos \beta}{\cos \alpha}$
- (3) $\frac{\cos^2 \beta}{\cos^2 \alpha}$
- (4) $\frac{\sin^2 \beta}{\cos^2 \alpha}$

Answer (3)

Sol. $v_1 \cos \alpha = v_2 \cos \beta$

$$\frac{K_1}{K_2} = \frac{v_1^2}{v_2^2} = \frac{\cos^2 \beta}{\cos^2 \alpha}$$



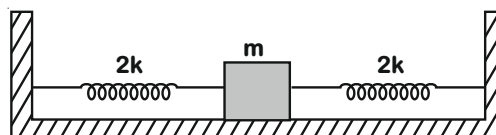
17. In a ferromagnetic material, below the curie temperature, a domain is defined as :

- (1) a macroscopic region with randomly oriented magnetic dipoles.
- (2) a macroscopic region with consecutive magnetic dipoles oriented in opposite direction.
- (3) a macroscopic region with saturation magnetization.
- (4) a macroscopic region with zero magnetization.

Answer (3)

Sol. Domain is the region in which magnetic moment is aligned in such a way to produce saturated magnetization.

18. Two identical springs of spring constant ' $2k$ ' are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is :



- | | |
|-------------------------------|------------------------------|
| (1) $\pi\sqrt{\frac{m}{k}}$ | (2) $\pi\sqrt{\frac{m}{2k}}$ |
| (3) $2\pi\sqrt{\frac{m}{2k}}$ | (4) $2\pi\sqrt{\frac{m}{k}}$ |

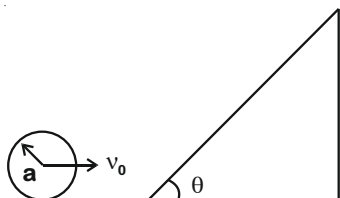
Answer (1)

Sol. $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$$k_{eq} = 4k$$

$$T = \pi\sqrt{\frac{m}{k}}$$

19. A sphere of radius 'a' and mass 'm' rolls along a horizontal plane with constant speed v_0 . It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping, how far up the sphere will travel?



- (1) $\frac{v_0^2}{5g \sin \theta}$ (2) $\frac{v_0^2}{2g \sin \theta}$
 (3) $\frac{10v_0^2}{7g \sin \theta}$ (4) $\frac{2 v_0^2}{5 g \sin \theta}$

Answer (3)

Sol. $K_T = \frac{1}{2} \times \frac{7}{5} mv_0^2$

$$\frac{7}{10} mv_0^2 = mg \ell \sin \theta$$

$$\ell = \frac{7v_0^2}{10g \sin \theta}$$

given answer in official option is $\frac{10v_0^2}{7g \sin \theta}$.

20. If e is the electronic charge, c is the speed of light in free space and h is Planck's constant,

the quantity $\frac{1}{4\pi\epsilon_0} \frac{|e|^2}{\hbar c}$ has dimensions of :

- (1) $[M L T^{-1}]$ (2) $[M L T^0]$
 (3) $[L C^{-1}]$ (4) $[M^0 L^0 T^0]$

Answer (4)

Sol. $[E] = \frac{hc}{\lambda}$

$$[E] = \frac{e^2}{4\pi\epsilon_0 r}$$

$$[M^0 L^0 T^0] = \frac{e^2}{4\pi\epsilon_0 r hc}$$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The wavelength of an X-ray beam is 10 \AA . The mass of a fictitious particle having the same energy as that of the X-ray photons is $\frac{x}{3} h \text{ kg}$. The value of x is _____.
 (h = Planck's constant)

Answer (10)

Sol. $E = \frac{hc}{\lambda} = mc^2$

$$\Rightarrow m = \frac{h}{\lambda c} = \frac{h}{10 \times 10^{-10} \times 3 \times 10^8} = \frac{10}{3} h \text{ kg}$$

2. Two identical conducting spheres with negligible volume have 2.1 nC and -0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m . The electrostatic force acting between the spheres is _____ $\times 10^{-9} \text{ N}$.

[Given: $4\pi\epsilon_0 = \frac{1}{9 \times 10^9} \text{ SI unit}$]

Answer (36)

Sol. $q'_1 = q'_2 = \frac{Q_1 + Q_2}{2} = 1 \text{ nC}$

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q'_1 q'_2}{r^2} = 9 \times 10^9 \times \frac{10^{-9} \times 10^{-9}}{(0.5)^2} = 36 \times 10^{-9} \text{ N}$$

3. The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4% will be _____ %.

Answer (2)

Sol. $v = \sqrt{\frac{T}{\mu}}$

$$\frac{\Delta v}{v} \times 100 = \frac{1}{2} \frac{\Delta T}{T} \times 100 = 2\%$$

4. The peak electric field produced by the radiation coming from the 8 W bulb at a

distance of 10 m is $\frac{x}{10} \sqrt{\frac{\mu_0 c}{\pi}} \frac{V}{m}$. The efficiency

of the bulb is 10% and it is a point source. The value of x is _____.

Answer (2)

Sol. $I = \frac{P}{4\pi r^2} = \frac{1}{2} \epsilon_0 E_0^2 \cdot c$

$$E_0 = \left(\frac{P}{2\pi\epsilon_0 c r^2} \right)^{\frac{1}{2}}$$

$$\frac{2}{10} \sqrt{\frac{\mu_0 c}{\pi}} \frac{V}{m}$$

5. If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, the angle between \vec{P} and \vec{Q} is θ ($0^\circ < \theta < 360^\circ$). The value of ' θ ' will be _____.

Answer (180)

Sol. $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$

$\Rightarrow \vec{P} \times \vec{Q} = 0$

$\Rightarrow \theta = 0^\circ$ or 180°

6. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of $10R$, where R is the radius of the earth, may be described in terms of

escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The

value of x will be _____.

Answer (10)

Sol. $-\frac{GM_e m}{R} + \frac{1}{2}mv^2 = -\frac{GM_e m}{11R}$

$v = \sqrt{\frac{20}{11} \frac{GM_e}{R}}$

$v_e = \sqrt{\frac{26GM_e}{R}}$

$v = \sqrt{\frac{10}{11}} \cdot v_e$

7. Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m . The charge on each of

the sphere is $\frac{a}{21} \times 10^{-8} \text{ C}$. The value of ' a ' will be.

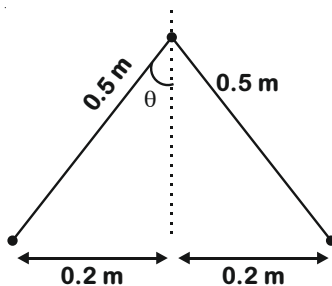
[Given $g = 10 \text{ ms}^{-2}$]

Answer (18.50)

Sol. $T \cos \theta = mg$

$T \sin \theta = \frac{q^2}{4\pi\epsilon_0(0.4)^2}$

$\tan \theta = \frac{q^2}{4\pi\epsilon_0(0.4)^2 mg}$



$\frac{0.20}{\sqrt{0.21}} = \frac{q^2 \times 9 \times 10^9}{(0.4)^2 \times 10^{-5}}$

$q^2 = \frac{0.16 \times 0.20}{9 \times 10^{14} \times \sqrt{0.21}}$

$q = \frac{0.4 \times 10^{-7}}{3} \times \sqrt{\frac{2}{\sqrt{21}}}$

$q = \frac{4}{3} \times \sqrt{\frac{2}{\sqrt{21}}} \times 10^{-8} \text{ C} = 0.88 \times 10^{-8} \text{ C}$

$a \approx 18.50$

8. A reversible heat engine converts one-fourth of the heat input into work. When the temperature of the sink is reduced by 52 K , its efficiency is doubled. The temperature in Kelvin of the source will be _____.

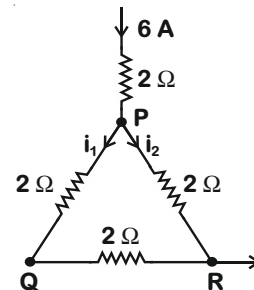
Answer (208)

Sol. $\eta = \frac{W}{Q} = 1 - \frac{T_L}{T_H} = \frac{1}{4} \Rightarrow T_L = \frac{3}{4} T_H$

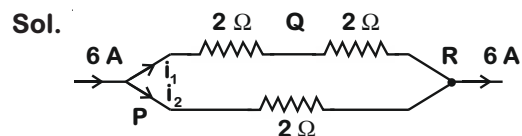
$\eta' = 1 - \frac{T_L - 52}{T_H} = \frac{1}{2} \Rightarrow T_L = \frac{1}{2} T_H + 52$

$T_H = 208 \text{ K}$

9. A current of 6 A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2Ω each and leaves by the corner R . The currents i_1 in ampere is _____.



Answer (2)



$4i_1 = 2i_2 \dots (i)$

$i_1 + i_2 = 6 \dots (ii)$

$\Rightarrow i_1 = 2 \text{ A}$

10. Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their momentum is $n : 2$. The value of n will

Answer (1)

Sol. $P = \sqrt{2mk}$

$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{16}} = \frac{1}{2}$

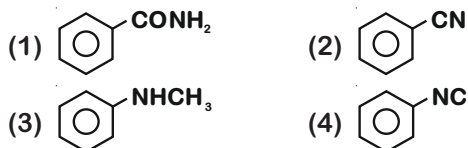
PART-B : CHEMISTRY

SECTION - I

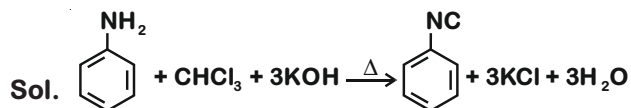
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline?

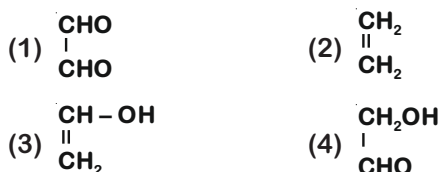
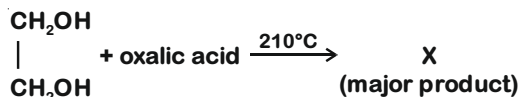


Answer (4)

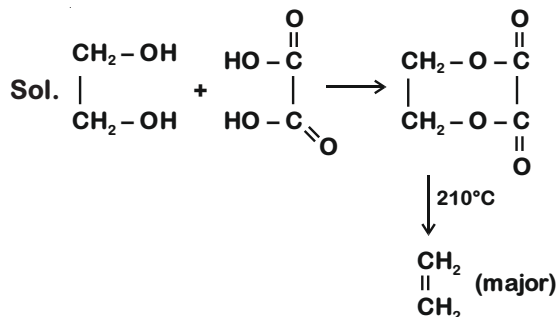


aniline

2. What is 'X' in the given reaction?



Answer (2)



3. The major components of German Silver are:



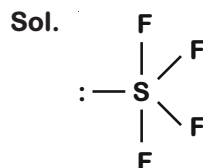
Answer (1)

Sol. German silver contains Cu (50%), Zn (30%), Ni (20%) respectively.

4. Which among the following species has unequal bond lengths?

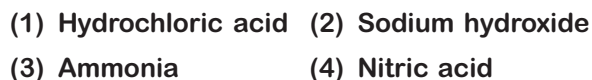


Answer (3)



axial bonds are longer than equatorial bonds. Only SF₄ has unequal bond length.

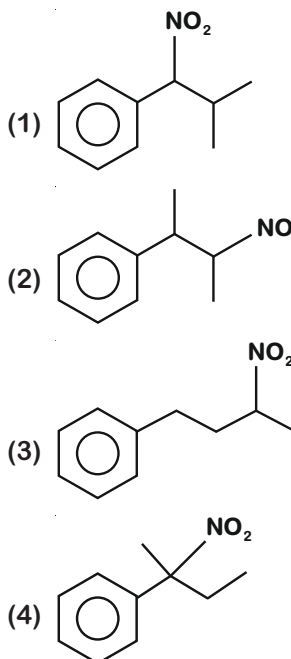
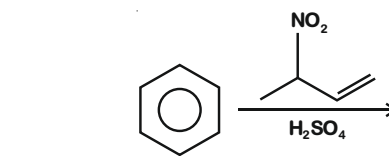
5. Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens?



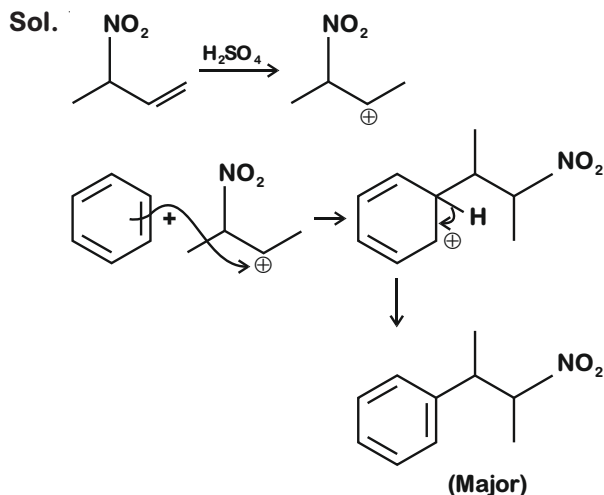
Answer (4)

Sol. The sodium fusion extract is acidified with nitric acid and then treated with silver nitrate.

6. The major product of the following reaction is:



Answer (2)



7. In which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment?

- (i) $[\text{FeF}_6]^{3-}$
- (ii) $[\text{Co}(\text{NH}_3)_6]^{3+}$
- (iii) $[\text{NiCl}_4]^{2-}$
- (iv) $[\text{Cu}(\text{NH}_3)_4]^{2+}$

- (1) (iii) > (iv) > (ii) > (i)
- (2) (ii) > (iii) > (i) > (iv)
- (3) (i) > (iii) > (iv) > (ii)
- (4) (ii) > (i) > (iii) > (iv)

Answer (3)

Sol.		Unpaired $e^-(n)$
(i) FeF_6^{3-}	Fe^{3+} (W.F.L)	5
(ii) $[\text{Co}(\text{NH}_3)_6]^{3+}$	Co^{3+} (S.F.L)	0
(iii) $[\text{NiCl}_4]^{2-}$	Ni^{2+} (W.F.L)	2
(iv) $[\text{Cu}(\text{NH}_3)_4]^{2+}$	Cu^{2+}	1

$$\mu = \sqrt{n(n+2)} \text{ B.M}$$

So, correct order of spin only magnetic moment is

$$(ii) < (iv) < (iii) < (i)$$

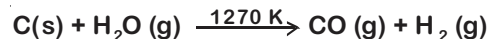
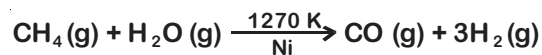
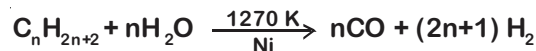
8. Water does not produce CO on reacting with:

- (1) C
- (2) CH_4
- (3) CO_2
- (4) C_3H_8

Answer (3)



all other will produce CO on reaction with water.



9. The correct order of bond dissociation enthalpy of halogens is:

- (1) $\text{Cl}_2 > \text{Br}_2 > \text{F}_2 > \text{I}_2$
- (2) $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$
- (3) $\text{Cl}_2 > \text{F}_2 > \text{Br}_2 > \text{I}_2$
- (4) $\text{I}_2 > \text{Br}_2 > \text{Cl}_2 > \text{F}_2$

Answer (1)

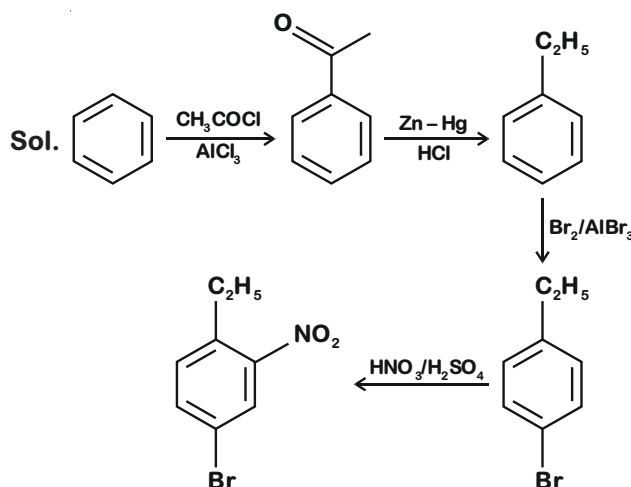


Bond dissociation enthalpy of F_2 is lower than Cl_2 and Br_2 . It is done to presence of e^- on fluorine atom, which create greater repulsion due to small size of fluorine.

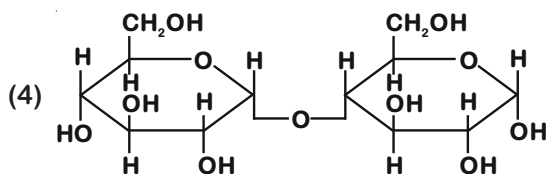
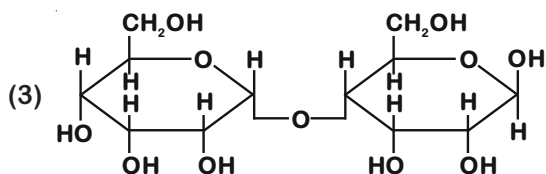
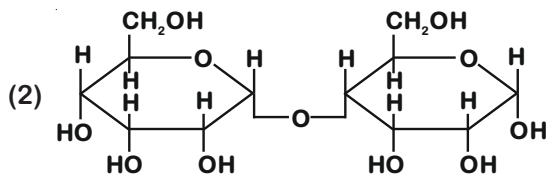
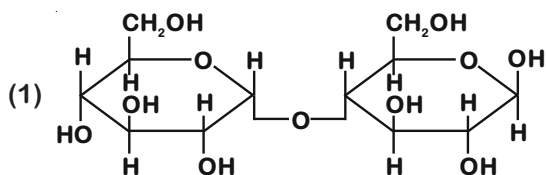
10. The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benzene is:

- (1) $\text{CH}_3\text{COCl}/\text{AlCl}_3$, $\text{Zn-Hg}/\text{HCl}$, $\text{Br}_2/\text{AlBr}_3$, $\text{HNO}_3/\text{H}_2\text{SO}_4$
- (2) $\text{HNO}_3/\text{H}_2\text{SO}_4$, $\text{Br}_2/\text{AlCl}_3$, $\text{CH}_3\text{COCl}/\text{AlCl}_3$, $\text{Zn-Hg}/\text{HCl}$
- (3) $\text{CH}_3\text{COCl}/\text{AlCl}_3$, $\text{Br}_2/\text{AlBr}_3$, $\text{HNO}_3/\text{H}_2\text{SO}_4$, Zn/HCl
- (4) $\text{Br}_2/\text{AlBr}_3$, $\text{CH}_3\text{COCl}/\text{AlCl}_3$, $\text{HNO}_3/\text{H}_2\text{SO}_4$, Zn/HCl

Answer (1)

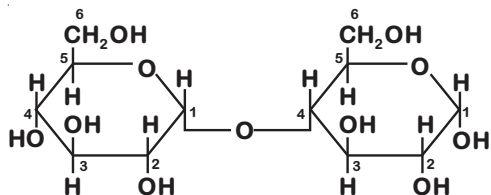


11. Which of the following is correct structure of α -anomer of maltose ?



Answer (4)

Sol. Maltose is composed of two units of α -D glucose which are joined through $C_1 - C_4$ glycosidic linkage



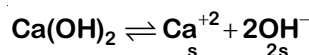
12. The solubility of $\text{Ca}(\text{OH})_2$ in water is :

[Given : The solubility product of $\text{Ca}(\text{OH})_2$ in water = 5.5×10^{-6}]

- (1) 1.77×10^{-2}
- (2) 1.11×10^{-2}
- (3) 1.77×10^{-6}
- (4) 1.11×10^{-6}

Answer (2)

Sol. Let s be the solubility of $\text{Ca}(\text{OH})_2$ in water



$$K_{sp} = [\text{Ca}^{+2}] [\text{OH}^-]^2$$

$$= s \times (2s)^2$$

$$5.5 \times 10^{-6} = 4s^3$$

$$s^3 = \frac{5.5}{4} \times 10^{-6} = 1.375 \times 10^{-6}$$

$$s = (1.375 \times 10^{-6})^{\frac{1}{3}}$$

$$= 1.11 \times 10^{-2}$$

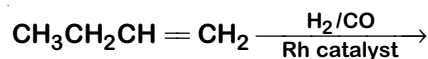
13. Which one of the following statements is FALSE for hydrophilic sols ?

- (1) Their viscosity is of the order of that of H_2O
- (2) They do not require electrolytes for stability
- (3) These sols are reversible in nature
- (4) The sols cannot be easily coagulated

Answer (1)

Sol. The viscosity of the hydrophilic sols are much higher than that of the dispersion medium.

14. The major product of the following reaction is :



- (1) $\text{CH}_3\text{CH}_2\underset{\text{CHO}}{\text{C}}=\text{CH}_2$
- (2) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CHO}$
- (3) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{CHO}$
- (4) $\text{CH}_3\text{CH}_2\text{CH}=\text{CH}-\text{CHO}$

Answer (3)

Sol. $\text{CH}_3\text{CH}_2\text{CH}=\text{CH}_2 \xrightarrow[\text{Rh catalyst}]{\text{H}_2/\text{CO}} \text{CH}_3\text{CH}_2\text{CH}_2-\text{CH}_2-\text{CHO}$

15. Given below are two statements :

Statement I : The identification of Ni^{2+} is carried out by Dimethylglyoxime in the presence of NH_4OH .

Statement II : The Dimethylglyoxime is a bidentate neutral ligand.

In the light of the above statements, choose the correct answer from the options given below :

- (1) Both Statement I and Statement II are true
- (2) Statement I is true but Statement II is false
- (3) Both Statement I and Statement II are false
- (4) Statement I is false but Statement II is true

Answer (2)

19. Given below are two statements :

Statement I :

The pH of rain water is normally ~5.6.

Statement II :

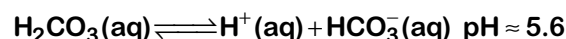
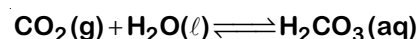
If the pH of rain water drops below 5.6, it is called acid rain.

In the light of the above statements, choose the correct answer from the options given below:

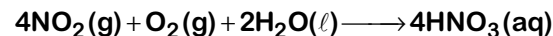
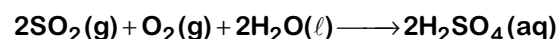
- (1) Both Statement I and Statement II are false.
- (2) Statement I is false but Statement II is true.
- (3) Both Statement I and Statement II are true.
- (4) Statement I is true but Statement II is false.

Answer (3)

Sol. pH of rain water is normally ~ 5.6 due to presence of H^+ formed by the reaction of water and CO_2 present in atmosphere



The pH of acid rain drop below 5.6 due to presence of other acidic gases like SO_2 and NO_2 present in atmosphere



$$pH < 5.6$$

20. Given below are two statements :

Statement I :

α and β forms of sulphur can change reversibly between themselves with slow heating or slow cooling.

Statement II :

At room temperature the stable crystalline form of sulphur is monoclinic sulphur.

In the light of the above statements, choose the correct answer from the options given below:

- (1) Both Statement I and Statement II are true.
- (2) Both Statement I and Statement II are false.
- (3) Statement I is true but Statement II is false.
- (4) Statement I is false but Statement II is true.

Answer (3)

Sol. The stable form at room temperature is rhombic sulphur, which transformed to monoclinic sulphur on heating at 369 K.

α and β form of sulphur can change reversibly between themselves with slow heating or slow cooling.

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The unit cell of copper corresponds to a face centered cube of edge length 3.596 Å with one copper atom at each lattice point. The calculated density of copper in kg/m^3 is ____.

[Molar mass of Cu : 63.54 g; Avogadro Number = 6.022×10^{23}]

Answer (9077)

Sol. Copper crystallises in fcc unit cell with edge length, $a = 3.596 \text{ \AA}$

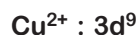
$$\begin{aligned} \text{density} &= \frac{4 \times M}{N_A (a)^3} \\ &= \frac{4 \times 63.54 \times 10^{-3}}{6.022 \times 10^{23} (3.596 \times 10^{-10})^3} \\ &\approx 9077 \text{ kg m}^{-3} \end{aligned}$$

2. The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is

Answer (2)

Sol. The element having atomic no. 29 is copper

The electronic configuration of Cu^{2+} is



It has 1 unpaired electron

$$\mu = \sqrt{3} = 1.73 \text{ BM} \approx 2$$

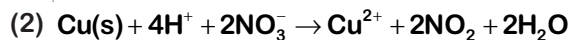
3. Copper reduces NO_3^- into NO and NO_2 depending upon the concentration of HNO_3 in solution. (Assuming fixed $[Cu^{2+}]$ and $P_{NO} = P_{NO_2}$), the HNO_3 concentration at which the thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same is 10^x M. The value of $2x$ is _____. (Rounded-off to the nearest integer)

[Given, $E_{Cu^{2+}/Cu}^\circ = 0.34 \text{ V}$, $E_{NO_3^-/NO}^\circ = 0.96 \text{ V}$, $E_{NO_3^-/NO_2}^\circ$

$$= 0.79 \text{ V and at } 298 \text{ K, } \frac{RT}{F} (2.303) = 0.059]$$

Answer (*)

Incomplete data.



Let $[\text{HNO}_3]$ be x , so $[\text{H}^+] = [\text{NO}_3^-] = x$

$$\Delta G_1 = -nFE_{\text{cell}} = -6F[E_{\text{HNO}_3/\text{NO}} - E_{\text{Cu}^{2+}/\text{Cu}}]$$

$$= -6F \left[(0.96 - 0.34) - \frac{0.059}{6} \log \frac{(\text{P}_{\text{NO}})^2 [\text{Cu}^{2+}]^3}{(x)^{10}} \right]$$

$$= -6F \left[0.62 - \frac{0.059}{6} \log \frac{(\text{P}_{\text{NO}})^2 [\text{Cu}^{2+}]^3}{x^{10}} \right]$$

$$\Delta G_2 = -2F \left[(0.79 - 0.34) - \frac{0.059}{2} \log \frac{(\text{P}_{\text{NO}_2})^2 [\text{Cu}^{2+}]}{x^6} \right]$$

$$= -2F \left[0.45 - \frac{0.059}{2} \log \frac{(\text{P}_{\text{NO}_2})^2 [\text{Cu}^{2+}]}{x^6} \right]$$

$$\Delta G_1 = \Delta G_2$$

$$-6F \left[0.62 - \frac{0.059}{6} \log \frac{(\text{P}_{\text{NO}})^2 [\text{Cu}^{2+}]^3}{x^{10}} \right]$$

$$= -2F \left[0.45 - \frac{0.059}{2} \log \frac{(\text{P}_{\text{NO}_2})^2 [\text{Cu}^{2+}]}{x^6} \right]$$

$$\left[1.86 - \frac{0.059}{2} \log \frac{(\text{P}_{\text{NO}})^2 [\text{Cu}^{2+}]^3}{x^{10}} \right]$$

$$= 0.45 - \frac{0.059}{2} \log \frac{(\text{P}_{\text{NO}_2})^2 [\text{Cu}^{2+}]}{x^6}$$

$$1.41 = \frac{0.059}{2} \left[\log \frac{(\text{P}_{\text{NO}})^2 [\text{Cu}^{2+}]^3}{x^{10}} - \frac{x^6}{(\text{P}_{\text{NO}_2})^2 [\text{Cu}^{2+}]} \right]$$

$$\frac{1.41 \times 2}{.059} = \log \frac{[\text{Cu}^{2+}]^2}{x^4}$$

$$\log \frac{[\text{Cu}^{2+}]}{x^2} = 23.9$$

4. The rate constant of a reaction increases by five times on increase in temperature from 27°C to 52°C . The value of activation energy in kJ mol^{-1} is _____. (Rounded-off to the nearest integer)

$$[R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}]$$

Answer (52)

$$\text{Sol. } \log \frac{K_2}{K_1} = \frac{E_a}{2.303 R} \left[\frac{1}{T_1} - \frac{1}{T_2} \right]$$

$$\log 5 = \frac{E_a}{2.303 \times 8.314} \left[\frac{1}{300} - \frac{1}{325} \right]$$

$$E_a = \frac{0.7 \times 2.303 \times 8.314 \times 300 \times 325}{25} \text{ J mol}^{-1}$$

$$= 52271 \text{ J} = 52.271 \text{ kJ mol}^{-1} \approx 52$$

5. Among the following, number of metal/s which can be used as electrodes in the photoelectric cell is _____. (Integer answer)

- (A) Li (B) Na
(C) Rb (D) Cs

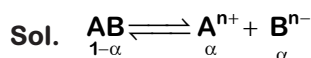
Answer (1)

Sol. Among the given alkali metals, only cesium (Cs) is used as electrode in the photoelectric cell due to its lowest ionisation energy.

6. If a compound AB dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5 K rise in the boiling point of the solution is ____ molal. (Rounded-off to the nearest integer)

$$[K_b = 0.52 \text{ K kg mol}^{-1}]$$

Answer (3)



$$i = 1 + \alpha = 1 + 0.75 (\because \alpha = 0.75)$$

$$= 1.75$$

$$\Delta T_b = i K_b m$$

$$m = \frac{2.5}{1.75 \times 0.52} = 2.75 \text{ mol/kg} \approx 3$$

7. Electromagnetic radiation of wavelength 663 nm is just sufficient to ionise the atom of metal A. The ionization energy of metal A in kJ mol^{-1} is _____. (Rounded-off to the nearest integer)

$$[h = 6.63 \times 10^{-34} \text{ Js}, c = 3.00 \times 10^8 \text{ ms}^{-1}, N_A = 6.02 \times 10^{23} \text{ mol}^{-1}]$$

Answer (181)

Sol. Ionisation energy of an atom of metal A = Quantum energy of radiation of wavelength 663 nm

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{663 \times 10^{-9}} \text{ J} = 3 \times 10^{-19} \text{ J}$$

Ionisation energy per mol

$$= 3 \times 10^{-19} \times 10^{-3} \times 6.02 \times 10^{23}$$

$$= 180.6 \text{ kJ mol}^{-1} \approx 181$$

8. Consider titration of NaOH solution versus 1.25 M oxalic acid solution. At the end point following burette readings were obtained.

- (i) 4.5 mL (ii) 4.5 mL
(iii) 4.4 mL (iv) 4.4 mL
(v) 4.4 mL

If the volume of oxalic acid taken was 10.0 mL then the molarity of the NaOH solution is ____ M. (Rounded-off to the nearest integer)

Answer (6)

Sol. Average volume of NaOH solution used at end point = 4.44 mL

At the end point, Equivalents of NaOH = Equivalents of oxalic acid

$$N_1 V_1 = N_2 V_2$$

$$N_1 \times 4.44 = (1.25 \times 2) \times 10$$

$$N_1 = \frac{1.25 \times 2 \times 10}{4.44} = 5.63 \approx 6$$

Molarity of NaOH = 6 M

9. Five moles of an ideal gas at 293 K is expanded isothermally from an initial pressure of 2.1 MPa to 1.3 MPa against at constant external pressure 4.3 MPa. The heat transferred in this process is ____ kJ mol^{-1} . (Rounded-off to the nearest integer)

$$[\text{Use } R = 8.314 \text{ J mol}^{-1} \text{K}^{-1}]$$

Answer (15)

Sol. For isothermal expansion at constant pressure, heat gained by the system is given by

$$Q = -W = + P_{\text{ex}} (V_2 - V_1)$$

$$= 4.3 \times 5 \times 8.314 \times 293 \left[\frac{1}{1.3} - \frac{1}{2.1} \right]$$

$$= 15.345 \text{ kJ mol}^{-1} \approx 15 \text{ kJ mol}^{-1}$$

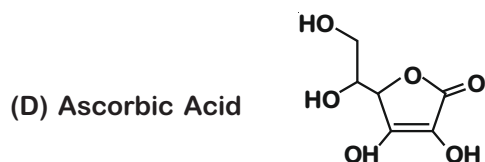
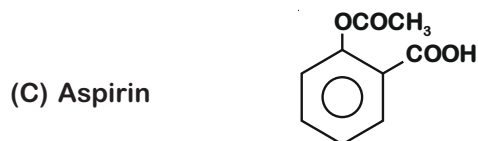
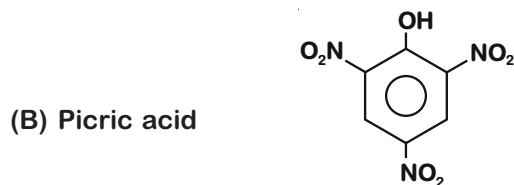
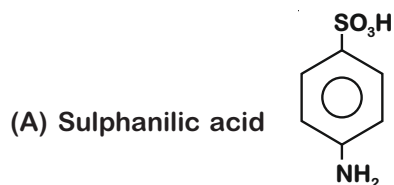
Note : The question seems to be ambiguous as the pressure of gas decreases from 2.1 MPa to 1.3 MPa at a constant pressure of 4.3 MPa which is much higher than the initial and pressure of the gas

10. The number of compound/s given below which contain/s $-\text{COOH}$ group is _____. (Integer answer)

- (A) Sulphanilic acid (B) Picric acid
(C) Aspirin (D) Ascorbic Acid

Answer (1)

Sol. The structures of the given compounds are



∴ Only 1 compound has $-\text{COOH}$ group

PART-C : MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x dx$, then :

(1) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(2) $I_2+I_4, I_3+I_5, I_4+I_6$ are in A.P.

(3) $I_2+I_4, (I_3+I_5)^2, I_4+I_6$ are in G.P.

(4) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

Answer (4)

Sol. $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{n-2} x (\cot^2 x) dx$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{n-2} x (\operatorname{cosec}^2 x - 1) dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{n-2} x \operatorname{cosec}^2 x dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{n-2} x dx$$

$$= -\frac{\cot^{n-1} x}{n-1} \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} - I_{n-2} = \frac{1}{n-1} - I_{n-2}$$

$$I_n + I_{n-2} = \frac{1}{n-1} \Rightarrow \frac{1}{I_{n-2} + I_n} = n-1 = \text{a linear expression in } n.$$

\therefore Sequence $\frac{1}{I_{n-2} + I_n}$ is an A.P.

2. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is :

(1) 2 (2) 4

(3) 3 (4) 1

Answer (1)

Sol. α, β are roots of $x^2 - 6x - 2 = 0$

$$\therefore \alpha^2 - 6\alpha - 2 = 0$$

$$\Rightarrow \alpha^2 - 2 = 6\alpha$$

Similarly $\beta^2 - 2 = 6\beta$

$$\frac{a_{10} - 2a_8}{3a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{(\alpha^{10} - 2\alpha^8) - (\beta^{10} - 2\beta^8)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)} = \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)} = 2$$

3. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is :

(1) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (2) $x^2 - y^2 = 9$

(3) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (4) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

Answer (1)

Sol. Eccentricity of Ellipse $e_1 = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Foci = $(\pm ae, 0) = (\pm 3, 0)$

For Hyperbola

Eccentricity $e_2 = \frac{5}{3}$

Semi-transverse axis $\rightarrow a = 3$

$$b^2 = a^2(e^2 - 1) = 9\left(\frac{25}{9} - 1\right) = 16$$

Equation of Hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

4. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$. Then :

- (1) $2y = 273x$ (2) $2y = 91x$
(3) $y = 273x$ (4) $y = 91x$

Answer (2)

Sol. $n(A) = 3, n(B) = 5$

$$x = {}^5C_3 \times 3! = 5 \times 4 \times 3$$

$$n(A \times B) = 15$$

$$y = {}^{15}C_3 \times 3! = 15 \times 14 \times 13$$

$$\frac{y}{x} = \frac{15 \times 14 \times 13}{5 \times 4 \times 3} = \frac{91}{2}$$

$$2y = 91x$$

5. A function $f(x)$ is given by $f(x) = \frac{5^x}{5^x + 5}$, then the sum of the series

$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \dots + f\left(\frac{39}{20}\right)$ is equal to:

- (1) $\frac{19}{2}$
(2) $\frac{29}{2}$
(3) $\frac{49}{2}$
(4) $\frac{39}{2}$

Answer (4)

Sol. $f(2-x) = \frac{5^{2-x}}{5^{2-x} + 5} = \frac{5}{5 + 5^x}$

So $f(x) + f(2-x) = 1$

$$\begin{aligned} \sum_{r=1}^{39} f\left(\frac{r}{20}\right) &= \sum_{r=1}^{19} \left(f\left(\frac{r}{20}\right) + f\left(2 - \frac{r}{20}\right) \right) + f(1) \\ &= 19 + \frac{1}{2} = \frac{39}{2} \end{aligned}$$

6. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in \mathbb{R}$ and $a > 0$, is equal to :

- (1) $a + 1$ (2) $2\sqrt{a}$
(3) $a + \frac{1}{a}$ (4) $2a$

Answer (2)

Sol. $f(x) = a^{a^x} + \frac{a}{a^{a^x}}$

$$\therefore \frac{a^{a^x} + \frac{a}{a^{a^x}}}{2} \geq \sqrt{a^{a^x} \cdot \frac{a}{a^{a^x}}}$$

$$\Rightarrow f(x) \geq 2\sqrt{a}$$

$$f(x)_{\min} = 2\sqrt{a}$$

7. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q , then the angle subtended by the line segment PQ at the origin is :

- (1) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$
(3) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$ (4) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

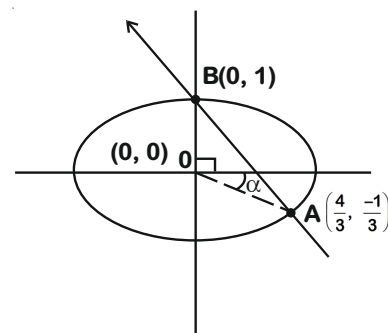
Answer (4)

Sol. $y = 1 - x$... (i)

$$x^2 + 2y^2 = 2$$
 ... (ii)

$$\Rightarrow x^2 + 2(1-x)^2 = 2$$

$$3x^2 - 4x = 0$$



$$x = 0, \frac{4}{3}$$

$$y = 1, -\frac{1}{3}$$

$$B(0,1), A\left(\frac{4}{3}, \frac{-1}{3}\right)$$

$$\tan \alpha = \frac{\frac{1}{3}}{\frac{4}{3}} = \frac{1}{4} \Rightarrow \alpha = \tan^{-1} \frac{1}{4}$$

$$\angle AOB = \frac{\pi}{2} + \tan^{-1} \frac{1}{4}$$

8. $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(2n-1)^2} \right]$

is equal to :

- (1) $\frac{1}{2}$ (2) $\frac{1}{3}$
 (3) 1 (4) $\frac{1}{4}$

Answer (1)

Sol. $\lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} \frac{n}{(n+r)^2}$
 $= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=0}^{n-1} \frac{1}{\left(1 + \frac{r}{n}\right)^2}$

$$\int_0^1 \frac{dx}{(1+x)^2} = -\frac{1}{1+x} \Big|_0^1 = -\frac{1}{2} + 1 = \frac{1}{2}$$

9. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is :

- (1) $\frac{2}{9}$ (2) $\frac{97}{297}$
 (3) $\frac{122}{297}$ (4) $\frac{1}{5}$

Answer (2)

Sol. Number having exactly one 7 can be

- (i) Having 7 at thousand's place = $9^3 = 729$
 (ii) Not 7 at thousand's place = $3 \times 8 \times 4^2 = 1944$
 $n(s) = 729 + 1944 = 2673$

Favourable cases = having 7 at unit place or having 2 at unit place.

$$\text{i.e.} = (9 \times 9) + (8 \times 9 \times 2) + (8 \times 9 \times 9) = 873$$

$$\text{Required probability} = \frac{873}{2673} = \frac{97}{297}$$

10. A plane passes through the points A(1,2,3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, -1, 1), then the projection of \overline{OP} on this plane is of length:

- (1) $\sqrt{\frac{2}{5}}$ (2) $\sqrt{\frac{2}{7}}$
 (3) $\sqrt{\frac{2}{3}}$ (4) $\sqrt{\frac{2}{11}}$

Answer (4)

Sol. $\overline{AB} = \hat{i} + \hat{j} - 2\hat{k}$

$$\overline{AC} = \hat{i} + 2\hat{j} - \hat{k}$$

Normal to plane $\vec{n} = \overline{AB} \times \overline{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$$\overline{OP} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\sin \theta = \frac{\overline{OP} \cdot \vec{n}}{|\overline{OP}| \times |\vec{n}|} = \frac{6+1+1}{\sqrt{11} \cdot \sqrt{6}} = \frac{8}{\sqrt{66}}$$

$$\cos \theta = \sqrt{1 - \frac{64}{66}} = \frac{1}{\sqrt{33}}$$

$$\text{Projection} = |\overline{OP}| \cos \theta = \sqrt{6} \times \frac{1}{\sqrt{33}} = \sqrt{\frac{2}{11}}$$

11. The integral $\int \frac{e^{3 \log_e 2x} + 5e^{2 \log_e 2x}}{e^{4 \log_e x} + 5e^{3 \log_e x} - 7e^{2 \log_e x}} dx$, $x > 0$, is equal to :

- (1) $4 \log_e |x^2 + 5x - 7| + c$
 (2) $\log_e |x^2 + 5x - 7| + c$
 (3) $\frac{1}{4} \log_e |x^2 + 5x - 7| + c$
 (4) $\log_e \sqrt{x^2 + 5x - 7} + c$

Answer (1)

Sol. $I = \int \frac{e^{3 \ln 2x} + 5e^{2 \ln 2x}}{e^{4 \ln x} + 5e^{3 \ln x} - 7e^{2 \ln x}} dx$

$$I = \int \frac{(2x)^3 + 5(2x)^2}{x^4 + 5x^3 - 7x^2} dx = \int \frac{8x + 20}{x^2 + 5x - 7} dx$$

Let $x^2 + 5x - 7 = t$

$$(2x + 5)dx = dt$$

$$I = 4 \int \frac{dt}{t} = 4 \ln |t| + c$$

$$I = 4 \ln |x^2 + 5x - 7| + c$$

$P(E/A_1) = 0.35, P(E/A_2) = 0.2, P(E/A_3) = 0.1$

to find $P(A_1/E)$

using Baye's theorem we get

$$P(A_1/E) = \frac{P(E/A_1) \cdot P(A_1)}{P(E/A_1) \cdot P(A_1) + P(E/A_2) \cdot P(A_2) + P(E/A_3) \cdot P(A_3)}$$

$$= \frac{0.35 \times 0.4}{(0.35 \times 0.4) + (0.2 \times 0.25) + (0.1 \times 0.35)}$$

$$= \frac{140}{140 + 50 + 35} = \frac{140}{225} = \frac{28}{45}$$

18. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

- (1) $\frac{1+\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{2}$
 (3) $\frac{1}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

Answer (1)

Sol. LHS = $\cos x + \cos y - \cos(x+y)$

$$= 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) - \left(2 \cos^2 \frac{x+y}{2} - 1\right)$$

$$\leq 2 \cos \frac{x+y}{2} - 2 \cos^2 \frac{x+y}{2} + 1$$

$$\therefore \left[\frac{x-y}{2} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow 0 < \cos\left(\frac{x-y}{2}\right) \leq 1 \right]$$

$$= 1 - 2 \left(\cos^2\left(\frac{x+y}{2}\right) - \cos\left(\frac{x+y}{2}\right) \right)$$

$$= 1 - 2 \left[\left(\cos\left(\frac{x+y}{2}\right) - \frac{1}{2} \right)^2 - \frac{1}{4} \right]$$

$$= \frac{3}{2} - 2 \left(\cos\left(\frac{x+y}{2}\right) - \frac{1}{2} \right)^2 \leq \frac{3}{2}$$

But given that LHS = $\frac{3}{2}$

$$\therefore \cos \frac{x-y}{2} = 1 \text{ and } \cos\left(\frac{x+y}{2}\right) = \frac{1}{2}$$

$$\Rightarrow x - y = 0 \text{ and } x + y = \frac{2\pi}{3}$$

$$\Rightarrow x = y = \frac{\pi}{3}$$

$$\Rightarrow \sin x + \cos y = \frac{\sqrt{3} + 1}{2}$$

19. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is :

- (1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$
 (3) 0 (4) $\frac{1}{2\sqrt{2}}$

Answer (4)

Sol. Equation of line parallel to $x - y = 1$ is

$$x - y = c \quad \dots(i)$$

If line $x - y = c$ is tangent to parabola $x^2 = 2y$ then $x^2 = 2(x - c)$ has unique roots

$$x^2 - 2x + 2c = 0$$

$$\therefore D = 0 \Rightarrow 4 - 4 \times 1 \times 2c = 0$$

$$\therefore c = \frac{1}{2}$$

\therefore Tangent of parabola is $x - y = \frac{1}{2}$

$$\therefore \text{Shortest distance} = \frac{\left|1 - \frac{1}{2}\right|}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \text{ units}$$

20. Let A be a 3×3 matrix with $\det(A) = 4$. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \rightarrow 2R_2 + 5R_3$ on 2A, then $\det(B)$ is equal to :

- (1) 64 (2) 128
 (3) 80 (4) 16

Answer (1)

Sol. Given $\det(A) = 4$

On application of $R_2 \rightarrow 2R_2 + 5R_3$ on 2A we have $2^3 \cdot 2 \det(A) = 16 \times 4 = 64$

SECTION - II

Numerical Value Type Questions: This section contains 10 questions. In Section II, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. If the curve, $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y) dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to _____.

Answer (01)

Sol. $\therefore (2xy^2 - y)dx + xdy = 0$

$$\Rightarrow 2xdx = \frac{ydx - xdy}{y^2}$$

$$\Rightarrow 2xdx = d\left(\frac{x}{y}\right)$$

On integrating both sides we get

$$x^2 = \frac{x}{y} + c \quad \dots(1)$$

The point eg intersection of lines $2x - 3y = 1$ and $3x + 2y = 8$ is $(2, 1)$

\therefore Curve (1) passes through $(2, 1)$ then $c = 2$

$$\therefore y(x) = \frac{x}{x^2 - 2}$$

$$\therefore y(1) = \frac{1}{1-2} = -1$$

$$\therefore |y(1)| = 1$$

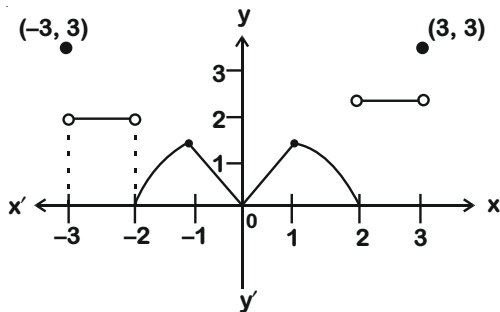
2. A function f is defined on $[-3, 3]$ as

$$f(x) = \begin{cases} \min\{|x|, 2-x^2\} & , -2 \leq x \leq 2 \\ [x] & , 2 < |x| \leq 3 \end{cases}$$

where $[x]$ denotes the greatest integer $\leq x$. The number of points, where f is not differentiable in $(-3, 3)$ is _____.

Answer (05)

Sol. $f(x) = \begin{cases} \min\{|x|, 2-x^2\} & , -2 \leq x \leq 2 \\ [x] & , 2 < |x| \leq 3 \end{cases}$



Now, $f(x) = \begin{cases} 3 & , x = -3 \\ 2 & , -3 < x < -2 \\ 2-x^2 & , -2 \leq x < -1 \\ -x & , -1 \leq x < 0 \\ x & , 0 \leq x < 1 \\ 2-x^2 & , 1 \leq x \leq 2 \\ 2 & , 2 < x < 3 \\ 3 & , x = 3 \end{cases}$

\therefore The points in $(-3, 3)$ where function is not differentiable is $x = -2, -1, 0, 1$ and 2 .

\therefore Total number of non differentiable points = 5

3. The value of $\int_{-2}^2 |3x^2 - 3x - 6| dx$ is _____.

Answer (19)

Sol. $\therefore 3x^2 - 3x - 6 = 3(x^2 - x - 2)$
 $= 3(x-2)(x+1)$

$$\begin{aligned} \therefore \int_{-2}^2 |3x^2 - 3x - 6| dx &= \int_{-2}^{-1} (3x^2 - 3x - 6) dx + \int_{-1}^2 (6 + 3x - 3x^2) dx \\ &= 3 \left\{ \int_{-2}^{-1} (x^2 - x - 2) dx + \int_{-1}^2 (2 + x - x^2) dx \right\} \\ &= 3 \left\{ \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_{-2}^{-1} + \left[2x + \frac{x^2}{2} - \frac{x^3}{3} \right]_{-1}^2 \right\} \\ &= 3 \left\{ \left(-\frac{1}{3} - \frac{1}{2} + 2 \right) - \left(-\frac{8}{3} - 2 + 4 \right) \right. \\ &\quad \left. + \left(4 + 2 - \frac{8}{3} \right) - \left(-2 + \frac{1}{2} + \frac{1}{3} \right) \right\} \\ &= 19 \end{aligned}$$

4. Let $\vec{a} = \hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - a\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors \vec{a} and \vec{b} is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to _____.

Answer (2)

Sol. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & 3 \\ 3 & -a & 1 \end{vmatrix} = 4\alpha\hat{i} + 8\hat{j} - 4\alpha\hat{k} = 4(\alpha\hat{i} + 2\hat{j} - \alpha\hat{k})$

$$\therefore 8\sqrt{3} = 4\sqrt{2\alpha^2 + 4} \Rightarrow \alpha = \pm 2$$

$$\vec{a} \cdot \vec{b} = 3 - \alpha^2 + 3 = 2$$

5. A line 'l' passing through origin is perpendicular to the lines

$$l_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$$

$$l_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$

If the co-ordinates of the point in the first octant on 'l₂' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l₁' are (a, b, c) then $18(a + b + c)$ is equal to _____.

Answer (44)

Sol. $l_1: \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2}$ and

$l_2: \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$$

So, $l: \frac{x}{-2} = \frac{y}{3} = \frac{z}{-2}$

Point of intersection of l and l_1 can be considered as

$P(-2\lambda, 3\lambda, -2\lambda)$ and $\frac{-2\lambda-3}{1} = \frac{3\lambda+1}{2} = \frac{-2\lambda-4}{2}$

$\Rightarrow P(2, -3, 2)$

Let a point Q on l_2 as $Q(2\mu + 3, 2\mu + 3, \mu + 2)$

$\therefore PQ = \sqrt{17} \Rightarrow (2\mu + 1)^2 + (2\mu + 6)^2 + \mu^2 = 17$

$\Rightarrow \mu = -\frac{10}{9}$ or -2

As Q lies in 1st octant, then $Q\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$,

Hence $18(a + b + c) = 44$

6. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is _____.

Answer (1)

Sol. $\therefore x = 4y + 3$

$$\begin{aligned} \text{then } (2020 + x)^{2022} &= (2023 + 4y)^{2022} \\ &= (4\lambda - 1)^{2022} \\ &= (16\lambda^2 - 8\lambda + 1)^{2022} \\ &= (8\mu + 1)^{1011} \\ &= 8\gamma + 1 \quad \text{where } \lambda, \mu, \gamma \in \mathbb{N} \end{aligned}$$

7. A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a + c)$ is equal to _____.

Answer (9)

Sol. Let equation of tangent to $y^2 = 4x$ as

$y = mx + \frac{1}{m}$

If it is a common tangent, then

$$\left| \frac{3m + \frac{1}{m}}{\sqrt{1 + m^2}} \right| = 3 \Rightarrow m = \pm \frac{1}{\sqrt{3}}$$

Equation of common tangent having point of contact in first quadrant ; $y = \frac{x+3}{\sqrt{3}}$.

The tangent intersects the parabola at $(3, 2\sqrt{3})$

and circle at $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

So, $2(a + c) = 9$

8. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is _____.

Answer (45)

Sol. $3^n + 7^n$ is divisible by $(3 + 7)$ if n is odd.

So, number of two digit odd numbers = 45

9. If $\lim_{x \rightarrow 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)}$ exists and is equal to b , then the value of $a - 2b$ is _____.

Answer (5)

Sol. $L = \lim_{x \rightarrow 0} \frac{a - \left(\frac{e^{4x} - 1}{x}\right)}{a(e^{4x} - 1)}$

$$L = \lim_{x \rightarrow 0} \frac{a - \frac{1}{x} \left[\frac{4x}{1} + \frac{(4x)^2}{2} + \dots \right]}{a \left[\frac{4x}{1} + \frac{(4x)^2}{2} + \dots \right]}$$

Clearly, $a - 4 = 0 \Rightarrow a = 4$

$L = \frac{-8}{16} = \frac{-1}{2} = b$

So, $a - 2b = 4 + 1 = 5$

10. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _____.

Answer (4)

Sol. $C_1: y^4 = x$ and $C_2: xy = k$

Point of intersection of C_1 and C_2 is $\left(k^{4/5}, k^{1/5}\right)$

$m_1 = \frac{dy_1}{dx} = \frac{1}{4y^3} = \frac{1}{4k^{3/5}}$

$m_2 = \frac{dy_2}{dx} = \frac{k}{x^2} = -\frac{1}{k^{3/5}}$

$\therefore m_1 \cdot m_2 = -1 \Rightarrow \frac{1}{4k^{6/5}} = 1 \Rightarrow 4k^{6/5} = 1$

$\Rightarrow (4k)^6 = 4$