FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24th February, 2021)   TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS	TEST PAPER WITH ANSWER

SECTION-A

1. For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \to q)) \to \sim p$

(b) $((p \vee q) \wedge \sim p) \to q$

Then which of the following statements is correct?

(1) (a) and (b) both are not tautologies.

(2) (a) and (b) both are tautologies.

(3) (a) is a tautology but not (b).

(4) (b) is a tautology but not (a).

Official Ans. by NTA (2)

2. Let $a, b \in R$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line

$\dfrac{x-3}{7} = \dfrac{y-2}{5} = \dfrac{z-1}{-9}$ is $(20, b, -a -9)$, then $|a + b|$

is equal to :

(1) 88   (2) 86

(3) 84   (4) 90

Official Ans. by NTA (1)

3. The vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i} + \hat{j}+ \hat{k})=1$ and $\vec{r}.(\hat{i} - 2\hat{j})=-2$, and the point $(1, 0, 2)$ is :

(1) $\vec{r}.(\hat{i} + 7\hat{j}+3\hat{k}) = \dfrac{7}{3}$

(2) $\vec{r}.(3\hat{i} + 7\hat{j}+3\hat{k}) = 7$

(3) $\vec{r}.(\hat{i} + 7\hat{j}+3\hat{k}) = 7$

(4) $\vec{r}.(\hat{i} - 7\hat{j}+3\hat{k}) = \dfrac{7}{3}$

Official Ans. by NTA (3)

4. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

(1) (3, 13)   (2) (1, 5)

(3) (–2, 8)   (4) (2, 8)

Official Ans. by NTA (4)

5. The angle of elevation of a jet plane from a point A on the ground is $60°$. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to $30°$. If the jet plane is flying at a constant height, then its height is :

(1) $1800\sqrt{3}$ m   (2) $3600\sqrt{3}$ m

(3) $2400\sqrt{3}$ m   (4) $1200\sqrt{3}$ m

Official Ans. by NTA (4)

6. If $n \geq 2$ is a positive integer, then the sum of the series $^{n+1}C_2 + 2(^2C_2 + {}^3C_2 + {}^4C_2 +....+ {}^nC_2)$ is:

(1) $\dfrac{n(n-1)(2n+1)}{6}$ (2) $\dfrac{n(n+1)(2n+1)}{6}$

(3) $\dfrac{n(2n+1)(3n+1)}{6}$ (4) $\dfrac{n(n+1)^2(n+2)}{12}$

Official Ans. by NTA (2)

7. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as,

$$f(x) = \begin{cases} -55\,x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \leq x \leq 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let $A = \{\mathbf{x} \in \mathbf{R} : f \text{ is increasing}\}$. Then A is equal to :

(1) $(-\infty, -5) \cup (4, \infty)$

(2) $(-5, \infty)$

(3) $(-\infty, -5) \cup (-4, \infty)$

(4) $(-5, -4) \cup (4, \infty)$

Official Ans. by NTA (4)

8. Let f be a twice differentiable function defined on R such that $f(0) = 1$, $f'(0) = 2$ and $f'(x) \neq 0$ for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of $f(1)$ lies in the interval:

(1) (9, 12)

(2) (6, 9)

(3) (0, 3)

(4) (3,6)

Official Ans. by NTA (2)

9. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\dfrac{3\sqrt{3}}{2}, \dfrac{1}{2} \right)$?

(1) $x^2 + y^2 = 7$ (2) $y^2 = \dfrac{1}{6\sqrt{3}}x$

(3) $2x^2 - 18y^2 = 9$ (4) $x^2 + 9y^2 = 9$

Official Ans. by NTA (4)

10. The value of the integral, $\int_1^3 [x^2 - 2x - 2]\,dx$, where [x] denotes the greatest integer less than or equal to x, is :

(1) $-\sqrt{2} - \sqrt{3} + 1$ (2) $-\sqrt{2} - \sqrt{3} - 1$

(3) -5 (4) -4

Official Ans. by NTA (2)

11. A possible value of $\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8} \right)$ is :

(1) $\dfrac{1}{\sqrt{7}}$ (2) $2\sqrt{2} - 1$

(3) $\sqrt{7} - 1$ (4) $\dfrac{1}{2\sqrt{2}}$

Official Ans. by NTA (1)

12. The negative of the statement $\sim p \wedge (p \vee q)$ is

(1) $\sim p \vee q$

(2) $p \vee \sim q$

(3) $\sim p \wedge q$

(4) $p \wedge \sim q$

Official Ans. by NTA (2)

13. If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point $(1,2)$ and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are :

(1) $a = \dfrac{1}{2}$, $b = \dfrac{1}{2}$, $c = 1$

(2) $a = 1$, $b = 0$, $c = 1$

(3) $a = 1$, $b = 1$, $c = 0$

(4) $a = -1$, $b = 1$, $c = 1$

Official Ans. by NTA (3)

14. The area of the region :
$R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is :

(1) $11\sqrt{3}$ square units

(2) $12\sqrt{3}$ square units

(3) $9\sqrt{3}$ square units

(4) $6\sqrt{3}$ square units

Official Ans. by NTA (2)

15. If a curve $y = f(x)$ passes through the point $(1, 2)$ and satisfies $x\dfrac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)\,dx = \dfrac{62}{5}$?

(1) 5   (2) 10

(3) $\dfrac{62}{5}$ (4) $\dfrac{31}{5}$

Official Ans. by NTA (2)

16. Let f(x) be a differentiable function defined on [0, 2] such that f '(x) = f '(2 − x) for all x ∈ (0, 2), f(0) = 1 and f(2) = e². Then the value of $\int\limits_{0}^{2} f(x)\,dx$ is :

(1) 1 − e² (2) 1 + e²

(3) 2(1 − e²) (4) 2(1 + e²)

Official Ans. by NTA (2)

17. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations (A²B² − B²A²)X = O, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × 1 null matrix, has :

(1) no solution

(2) exactly two solutions

(3) infinitely many solutions

(4) a unique solution

Official Ans. by NTA (3)

18. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\dfrac{10}{3}, \dfrac{7}{3}\right)$. If α, β are the roots of the equation ax² + bx + 1 = 0, then the value of α² + β² − αβ is :

(1) $\dfrac{71}{256}$ (2) $\dfrac{69}{256}$

(3) $-\dfrac{69}{256}$ (4) $-\dfrac{71}{256}$

Official Ans. by NTA (4)

19. For the system of linear equations :

x − 2y = 1, x − y + kz = −2, ky + 4z = 6, k ∈ R, consider the following statements :

(A) The system has unique solution if k ≠ 2, k ≠ −2.

(B) The system has unique solution if k = −2.

(C) The system has unique solution if k = 2.

(D) The system has no-solution if k = 2.

(E) The system has infinite number of solutions if k ≠ −2.

Which of the following statements are correct ?

(1) (C) and (D) only

(2) (B) and (E) only

(3) (A) and (E) only

(4) (A) and (D) only

Official Ans. by NTA (4)

20. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is :

(1) $\dfrac{65}{2^7}$ (2) $\dfrac{65}{2^8}$

(3) $\dfrac{135}{2^9}$ (4) $\dfrac{35}{2^7}$

Official Ans. by NTA (3)

SECTION-B

1. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \ge r \ge 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$$ exists, is equal to _____.

Official Ans. by NTA (12)

Ans. by ALLEN (BONUS)

2. Let λ be an interger. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\dfrac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _____.

Official Ans. by NTA (1)

3. If $a + \alpha = 1$, $b + \beta = 2$ and

$af(x) + \alpha f\left(\dfrac{1}{x}\right) = bx + \dfrac{\beta}{x}$, $x \neq 0$, then the value

of expression $\dfrac{f(x) + f\left(\dfrac{1}{x}\right)}{x + \dfrac{1}{x}}$ is _____.

Official Ans. by NTA (2)

4. Let a point P be such that its distance from the point $(5, 0)$ is thrice the distance of P from the point $(-5, 0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to _____.

Official Ans. by NTA (56)

5. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x - 2)^2 + (y - 3)^2 = 25$ at the point $(5, 7)$ is A, then 24A is equal to_____.

Official Ans. by NTA (1225)

Ans. by ALLEN (1225 / BONUS)

6. If the variance of 10 natural numbers 1, 1, 1,...., 1, k is less than 10, then the maximum possible value of k is _____.

Official Ans. by NTA (11)

7. The sum of first four terms of a geometric progression (G.P.) is $\dfrac{65}{12}$ and the sum of their respective reciprocals is $\dfrac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is _____.

Official Ans. by NTA (3)

8. The students $S_1, S_2,....., S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

Official Ans. by NTA (31650)

9. Let $i = \sqrt{-1}$. If $\dfrac{(-1 + i\sqrt{3})^{21}}{(1-i)^{24}} + \dfrac{(1 + i\sqrt{3})^{21}}{(1+i)^{24}} = k$,

and $n = [|k|]$ be the greatest integral part of

$|k|$. Then $\displaystyle\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to

_____.

Official Ans. by NTA (310)

10. The number of the real roots of the equation

$(x + 1)^2 + |x - 5| = \dfrac{27}{4}$ is _____.

Official Ans. by NTA (2)