JEE-Main-26-02-2021-Shift-2 (Memory Based)

PHYSICS

Question: If a wire of length l has a resistance of R, is stretched by 25%. The percentage change in its resistance is?

Options:

(a) 25%

(b) 50%

(c) 45.25%

(d) 56.25%

Answer: (d)

Solution:

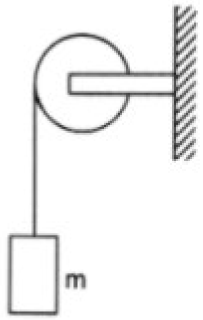
$$R = \rho\frac{l}{A} = \rho\frac{l^2}{V}\quad (\because V = Al)$$

$$R' = \rho\frac{(1.25)^2 l^2}{V} = 1.5625R$$

$$\%R = \left(\frac{R' - R}{R}\right)100 = (1.5625 - 1)\times 100$$

$$= 56.25\%$$

Question: A chord is tied to a wheel of moment of inertia I and radius r. The other end is attached to a mass ‘m’ as shown. If the mass ‘m’ falls by a height ‘h’ then the square of angular of speed of the wheel is?



Options:

(a) $\dfrac{mgh}{I + mr^2}$

(b) $\dfrac{2mgh}{I + mr^2}$

(c) $\dfrac{2mgh}{2I + mr^2}$

(d) $\dfrac{mgh}{2I + mr^2}$

Answer: (b)

Solution:

Considering no slipping between chord and wheel and considering no energy loss due to friction.
So, by Energy Conservation:-

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow mgh = \frac{1}{2}m(\omega r)^2 + \frac{1}{2}I\omega^2 \qquad (\because v = \omega r)$$

$$\Rightarrow \omega^2 = \frac{2mgh}{mr^2 + I}$$

Question: What is the recoil velocity of Hydrogen atom when a photon is emitted due to corresponding transition from n = 5 to n =1. (R = Rydberg's constant. m_H = mass of hydrogen atom)

Options:

(a) $\frac{hR}{m_H}$

(b) $\frac{hR}{25m_H}$

(c) $\frac{4hR}{25m_H}$

(d) $\frac{24hR}{25m_H}$

Answer: (d)

Solution:

Energy released during transition of e^- from n = 5 to n = 1

$$\Rightarrow E = E_5 - E_1 = Rhc\left(-\frac{1}{(5)^2} - \left(\frac{-1}{(1)^2}\right)\right)$$

$$\Rightarrow E = \frac{24}{25}Rhc \ldots(i)$$

So momentum of Photon released would be:-

$$\Rightarrow E = mc^2 = (mc).c = p.c$$

Using equation (i)

$$\Rightarrow p = \frac{E}{c} = \frac{24}{25}Rh$$

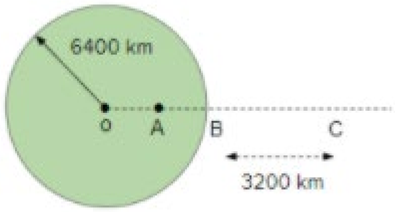
So Recoil velocity of H-atom would be:
By conservation of linear Momentum.

$$\Rightarrow m_H v_H = p = \frac{24}{25}Rh$$

$$v_H = \frac{24}{25}\frac{Rh}{m_H}$$

Question: For earth's gravitation

Given: $[g_A = g_C < g_B]$. Find $\frac{OA}{AB}$.



Options:
(a) 1 : 1
(b) 2 : 3
(c) 4 : 5
(d) 4 : 9
Answer: (c)
Solution:

$$\Rightarrow g_C = \frac{GM}{(R+R/2)^2} = \frac{4}{9}\frac{GM}{R^2} = \frac{4}{9}g$$

$$\Rightarrow g_A = g\frac{x}{R} \ (\text{where } x = OA)$$

So, if $g_A = g_C$

$$\Rightarrow \frac{4}{9}g = g\frac{x}{R}$$

$$\Rightarrow x = \frac{4}{9}R$$

To find :- $\frac{OA}{AB} = \frac{x}{R-x} = \frac{4}{5}$ (Where AB=OB-OA and OB=R)

Question: Find Dimension of $\frac{C}{V}$?

Options:
(a) $\left[M^{-2}L^{-4}T^{7}A^{3}\right]$
(b) $\left[M^{2}L^{4}T^{-6}A^{-2}\right]$
(c) $\left[M^{2}L^{-4}T^{6}A^{2}\right]$
(d) $\left[M^{-2}L^{4}T^{-6}A^{2}\right]$
Answer: (a)
Solution:

$$\frac{C}{V} = \frac{Q}{V^2} = \frac{Q}{(W/Q)^2} = \frac{Q^3}{W^2} = \left(\frac{It}{W^2}\right)^2$$

$$\left[\frac{C}{V}\right] = \frac{[It]^3}{[W]^2} = \frac{A^3T^3}{(ML^2T^{-2})^2}$$

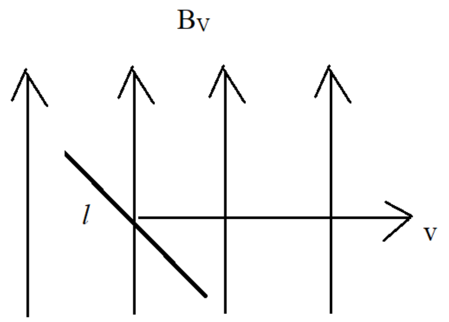
$$= M^{-2}L^{-4}T^{7}A^{3}$$

Question: An aeroplane with its wings spread 10 m is flying with speed 180 kph in horizontal direction. The total intensity of earth's field is 2.5×10^{-4} Tesla and angle of dip is $60°$. Then find emf induced between the tips of the plane wings.

Options:
(a) 108 mV
(b) 54 mV
(c) 216 mV
(d) 140 mV

Answer: (a)

Solution:



$B = 2.5\times10^{-4}\,T$

$\delta = 60^{\circ}$

$B_v = B\sin\delta$

$= 2.5\times10^{-4}\sin 60 = \frac{2.5\sqrt{3}}{2}\times10^{-4}\,T$

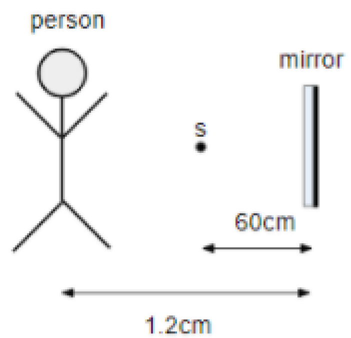
$l = 10\,m$

$V = 180\,km/h = \frac{180\times5}{18} = 50\,m/s$

$|E| = Bvlv = \frac{2.5\sqrt{3}}{2}\times10^{-4}\times10\times50$

$= 1082\times10^{-4}\,V = 108\times10^{-3}\,V = 108\,mV$

Question: A person walks parallel to a 50 cm wide plane mirror as shown. How much distance will he be able to see the image of a source placed 60 cm in point of it?

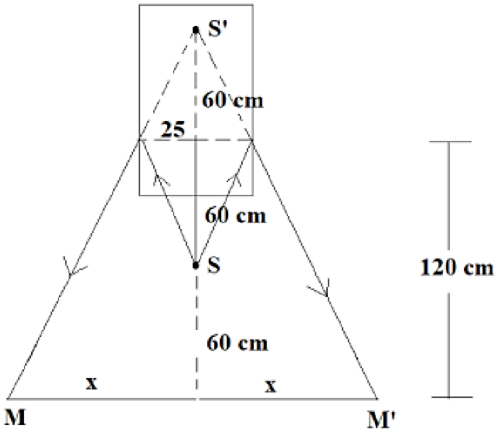


Options:
(a) 50 cm
(b) 100 cm
(c) 150 cm

(d) 200 cm

Answer: (c)

Solution:



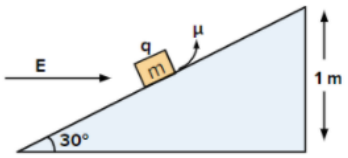
Man can see image by while traversing MM'

Now,

$$\frac{25}{60}=\frac{x}{180}\Rightarrow x=75$$

$$MM'=2x=150\,cm$$

Question: Find the time taken by the block to reach the bottom of inclined plane. E = 200 i N/C, M = 1 kg, q = 5 mC, g = 10 m/s^2, $\mu=0.2$



Options:

(a) 1.35 s

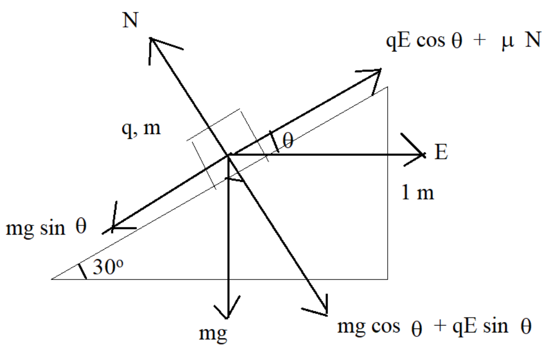
(b) 1.65 s

(c) 1.9 s

(d) 2.3 s

Answer: (a)

Solution:



Net force along the incline

$$F = mg\sin\theta - (\mu N + qE\cos\theta)$$
$$= mg\sin\theta - \mu(mg\cos\theta + qE\sin\theta) - qE\cos\theta$$
$$= 1\times10\sin30 - 0.2(1\times10\times\cos30 + 200\times5\times10^{-3}\times\sin30) - 200\times5\times10^{-3}\cos30$$
$$= 5 - 0.2(5\sqrt{3}+0.5) - \sqrt{3}/2$$
$$= 2.3N$$
$$a = \frac{F}{m} = \frac{2.3}{1} = 2.3\,m/s^2$$
Time taken to slide down 2 m long

Incline $t = \sqrt{\frac{25}{a}} = \sqrt{\frac{2\times2}{2.3}} = 1.32\,s$

Question: Statement 1: A seconds pendulum, has a time period of 1 second.
Statement 2: It takes precisely 1 second to move between the two extreme position.
Options:
(a) Statement 1 is false, Statement 2 is true
(b) Statement 1 is true, Statement 2 is true
(c) Statement 1 is true, Statement 2 is false
(d) Statement 1 is false, Statement 2 is false
Answer: (a)
Solution:
[Statement 1 is false, Statement 2 is true]
A **seconds pendulum** is a **pendulum** whose **period** is precisely two **seconds**; one **second** for a swing in one direction and one second for the return swing.
So it will take 1 second to move between two extreme positions.
Thus statement 1 is false and statement 2 is true.

Question: Velocity v/s position graph of a body performing SHM is
Options:
(a) ellipse
(b) circle
(c) parabola
(d) straight line
Answer: (a)
Solution:
For SHM

$x = A\sin\omega t$ ...(i)

$v = \frac{d(x)}{dt} = A\omega\cos\omega t$ ...(ii)

From equation (i)

$\sin\omega t = \frac{x}{A} \Rightarrow \sin^2\omega t = \frac{x^2}{A^2}$ ...(iii)

From equation (ii)

$\cos\omega t = \frac{v}{A\omega} \Rightarrow \cos^2\omega t = \frac{v^2}{A^2\omega^2}$ ...(iv)

Adding equation (iii) and (iv)

$$\sin^2 \omega t + \cos^2 \omega t = \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2}$$
$$\Rightarrow 1 = \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2}$$
This is clearly on equation of ellipse.

Question: A body starts from rest and moves with constant acceleration a, for time t_1, then it retards uniformly with a_2 in time t_2. Find t_1/t_2.
Options:
(a) $\frac{a_1}{a_2}$
(b) $\frac{a_2}{a_1}$
(c) 1
(d) None of these
Answer: (b)
Solution:
For acceleration period,
$u = 0,\ v = u,\ a = a_1,\ t = t_1$

So, $v = u = at \Rightarrow v = 0 + a_1 t_1 \Rightarrow t_1 = \frac{v}{a_1}$ ...(i)

For retardation period,
$u = v,\ v = 0,\ a = -a_2,\ t = t_2$

So, $v = u + at \Rightarrow 0 = v - a_2 t_2, \Rightarrow t_2 = \frac{v}{a_2}$ ...(ii)

On dividing equation (i) by (ii) $\rightarrow$
$$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$

Question: A wire has length l_1 when tension in it is T_1 & l_2 when tension is T_2. Find the natural length of wire.
Options:
(a) $\frac{T_1 l_1 - T_2 l_2}{T_1 - T_2}$
(b) $\frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$
(c) $\frac{T_1 l_1 + T_2 l_2}{T_1 + T_2}$
(d) $\frac{T_1 l_2 + T_2 l_1}{T_1 + T_2}$
Answer: (b)
Solution:
Let the natural length of wire be l_0.

Using Hooke's law, $Y=\frac{Tl_0}{A\Delta l}$

Where $\Delta l = l - l_0$

We get $l-l_0=\frac{Tl_0}{AY}$

Case 1: Tension T_1 and length of wire $l=l_1$

$\therefore l_1-l_0=\frac{T_1 l_0}{AY}....(1)$

Case 2: Tension is T_2 and length of wire $l=l_2$

$\therefore l_2-l_0=\frac{T_2 l_0}{AY}...(2)$

Dividing both equations $\frac{l_1-l_0}{l_2-l_0}=\frac{T_1}{T_2}$

$\Rightarrow l_0=\frac{l_1T_2-l_2T_1}{T_2-T_1}$

Question: A radioactive sample is undergoing α-decay. At time t_1, its activity is A and at another time t_2, the activity is $\frac{A}{5}$. What is the average life time for the sample

Options:

(a) $\frac{t_2-t_1}{\ln 2}$

(b) $(t_2-t_1)\ln 5$

(c) $\frac{t_2-t_1}{\ln 5}$

(d) $\frac{t_2-t_1}{2}$

Answer: (c)

Solution:

Activity $=\left|\frac{dN}{dt}\right|$

At time t_1 $\quad A=N_0\lambda e^{-\lambda t_1} \quad ...(1)$

At time t_2 $\quad \frac{A}{5}=N_0\lambda e^{-\lambda t_2} \quad ...(2)$

From eq (1) &(2)

$5=\frac{e^{-\lambda t_1}}{e^{-\lambda t_2}}$

$5=e^{-\lambda(t_1-t_2)}$

$\ln 5=-\lambda(t_1-t_2)$

$\ln 5=\lambda(t_2-t_1)$

$\lambda=\frac{\ln 5}{t_1-t_1}$

Mean lifetime given by $\tau = \frac{1}{\lambda}$

$\tau = \frac{t_2 - t_1}{\ln 5}$

Question: A bike starts from rest and accelerates uniformly at 'a' m/s^2 for time 't_1' seconds. Then it retards with deceleration 'a' for time 't_2' seconds with till it comes to rest. Find the average speed for the entire duration.

Options:

(a) $\frac{a(t_1 + t_2)}{2}$

(b) $\frac{at_2}{2}$

(c) $\frac{at_1^2}{2}$

(d) at_1

Answer: (b)

Solution:

Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

Initial speed is zero.

And acceleration is a.

$v = u + at$

$v = at_1$ after time t1

& $S_1 = \frac{1}{2} a t_1^2$

Now,

$v = u + at$

$o = at_1 - at_2$

$at_1 = at_2 \Rightarrow t_1 = t_2$

$S_2 = at_1 t_2 - \frac{1}{2} at_2^2$

Total distance $= S_1 + S_2$

$= \frac{1}{2} at_1^2 + at_1 t_2 - \frac{1}{2} at_2^2$

$= \frac{1}{2} at^2 + at^2 - \frac{1}{2} at^2$

$S = at^2$

$t_1 + t_2 = 2t$

$\langle v \rangle = \frac{at^2}{2t}$

$= \frac{at}{2} = \frac{at_2}{2}$

Question: If incident say, refracted ray and normal say are represented by unit vectors $\vec{a}, \vec{b}\, and\, \vec{c}$ then relation between them is?

Options:

(a) $\vec{a} - \vec{b} = \vec{c}$

(b) $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = 0$

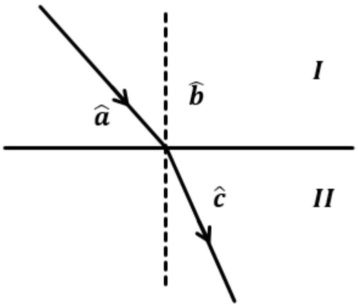
(c) $\vec{a} + \vec{c} = 2\vec{b}$

(d) $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = 0$

Answer: (b)

Solution:

Let $\mu_1 < \mu_2$



All three unit vectors are coplanar, we can say this from first law of refraction

Scalar triple product is given by $\vec{A}.\left(\vec{B} \times \vec{C}\right)$

If $\vec{A}, \vec{B}\ \&\ \vec{C}$ vectors are coplanar then

$\vec{A}.\left(\vec{B} \times \vec{C}\right) = 0 \qquad \ldots\text{(i)}$

From eq. (i) we have

$\hat{a}.\left(\hat{b} \times \hat{c}\right) = 0$

Question: If the internal energy of a gas is $U = 3PV + 4,$ then the gas can be?

Options:

(a) Monoatomic

(b) Diatomic

(c) Polyatomic

(d) Either mono or diatomic

Answer: (c)

Solution:

Given, $U = 3PV + 4$

We have $PV = nRT$

$U = 3\left(nRT\right) + 4$

Differentiating wrt temperature

$dU = 3.\left(nRdT\right) + 0$

$\frac{nfRdT}{2} = 3\left(nRdT\right)$

$\frac{f}{2} = 3 \Rightarrow f = 6$

It would be triatomic, suitable option is Polyatomic.

JEE-Main-26-02-2021-Shift-2 (Memory Based)

CHEMISTRY

Question: Increasing order of Δ_{eg} H of the following elements:
O, S, Se, Te (Consider both sign and magnitude)
Options:

(a) S < Se < Te < O

(b) O < S < Se < Te

(c) S < O < Se < Te

(d) O < Te < Se < S

Answer: (a)

Solution: The values are,

S = –200 kJ/mol

Se = –195 kJ/mol

Te = –190 kJ/mol

O = –141 kJ/mol

So, considering both sign and magnitude, the order should

S < Se < Te < O

Question: Hybridisation order of the carbon atom from left to right is
$CH_2{=}C{=}CH{-}CH_3$
Options:

(a) sp^2, sp, sp^2, sp^3

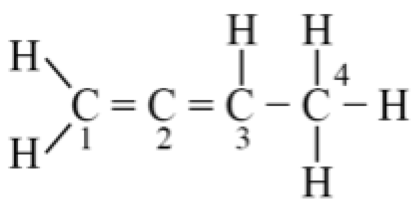
(b) sp^2, sp^2, sp^2, sp^3

(c) sp^2, sp, sp, sp^3

(d) sp, sp, sp^2, sp^3

Answer: (a)

Solution:



$1 - sp^2$

$2 - sp$

$3 - sp^2$

$4 - sp^3$

Question: Match the following

Column-I	**Column-II**
(A) Siderite	**(P)** Fe
(B) Calamine	**(Q)** Al
(C) Cryolite	**(R)** Zn
(D) Malachite	**(S)** Cu

Options:

(a) $A \to P$; $B \to R$; $C \to S$; $D \to Q$

(b) $A \to P$; $B \to S$; $C \to Q$; $D \to Q$

(c) $A \to P$; $B \to R$; $C \to Q$; $D \to S$

(d) $A \to Q$; $B \to R$; $C \to P$; $D \to S$

Answer: (c)

Solution:

Siderite ($FeCO_3$) is an ore of iron

Calamine ($ZnCO_3$) is an ore of zinc

Cryolite (Na_3AlF_6) is an ore of Aluminium

Malachite ($CuCO_3$, $Cu(OH)_2$) is an ore of copper.

Question: Which of the following groups contains both acidic oxides:
Options:

(a) N_2O, BaO

(b) CaO, SiO_2

(c) B_2O_3, SiO_2

(d) B_2O_3, CaO

Answer: (c)

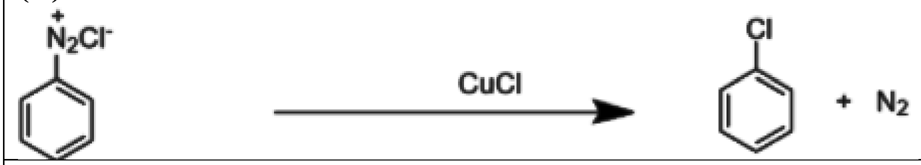
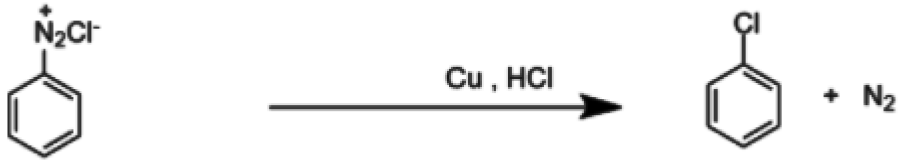
Solution:

$N_2O \to$ Neutral

BaO, CaO $\rightarrow$ Basic

B_2O_3, SiO_2 $\rightarrow$ Acidic

Question: Match the following.

Column-I	**Column-II**
(A) 	**(P)** Wurtz Reaction
(B) 	**(Q)** Sandmeyer Reaction
(C) $2CH_2CH_2Cl + 2Na \xrightarrow{\text{dry ether}} CH_3CH_2CH_2CH_3$	**(R)** Fittig Reaction
(D) 	**(S)** Gattermann Reaction

Options:

(a) A $\rightarrow$ P; B $\rightarrow$ Q; C $\rightarrow$ R; D $\rightarrow$ S

(b) A $\rightarrow$ Q; B $\rightarrow$ S; C $\rightarrow$ P; D $\rightarrow$ R

(c) A $\rightarrow$ Q; B $\rightarrow$ S; C $\rightarrow$ R; D $\rightarrow$ P

(d) A $\rightarrow$ S; B $\rightarrow$ Q; C $\rightarrow$ P; D $\rightarrow$ R

Answer: (b)

Solution: Sandmeyer takes place with Cu^+

Gattermann takes place with $Cu^{[O]}$

Alkyl halide coupling is Wurtz

Aryl halide coupling is Fittig Reaction

Question: Match the following.

Molecule	Bond order
(A) Ne_2	**(P)** 1
(B) N_2	**(Q)** 2
(C) F_2	**(R)** 0
(D) O_2	**(S)** 3

Options:

(a) $A \rightarrow R$; $B \rightarrow S$; $C \rightarrow Q$; $D \rightarrow P$

(b) $A \rightarrow R$; $B \rightarrow S$; $C \rightarrow P$; $D \rightarrow Q$

(c) $A \rightarrow R$; $B \rightarrow P$; $C \rightarrow Q$; $D \rightarrow S$

(d) $A \rightarrow R$; $B \rightarrow Q$; $C \rightarrow P$; $D \rightarrow P$

Answer: (b)

Solution: $B.O = \dfrac{\text{No. of bonding e}^- - \text{No. of antibonding e}^-}{2}$

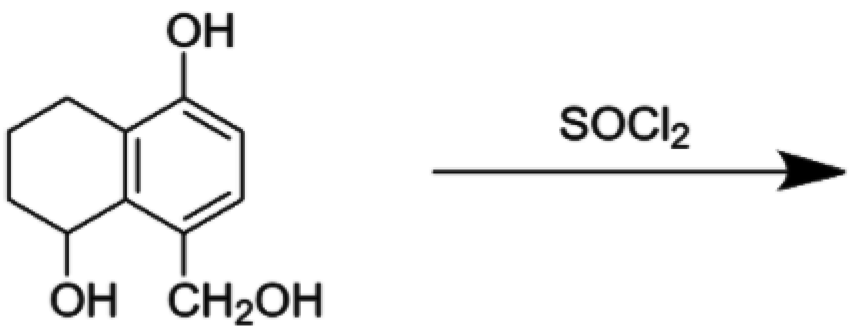
a) $Ne_2 = \dfrac{10-10}{2} = 0$

b) $N_2 = \dfrac{10-4}{2} = 3$

c) $F_2 = \dfrac{10-8}{2} = 1$

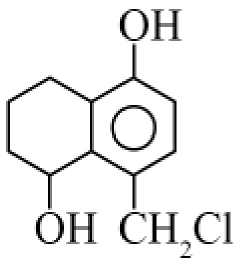
d) $O_2 = \dfrac{10-6}{2} = 2$

Question: Final product of the reaction

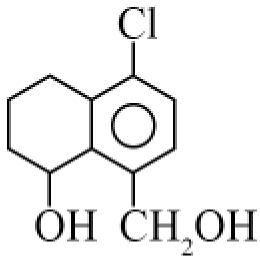


Options:

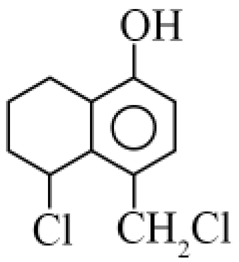
(a)



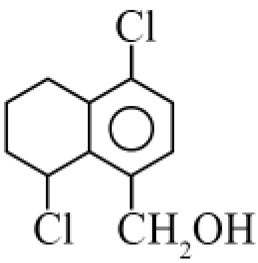
(b)



(c)



(d)



Answer: (c)

Solution: Allylic position is reactive for nucleophilic substitution reaction

Question: False statement about Calgon is:

Options:

(a) Calgon is also called as graham's salt

(b) Calgon method does not precipitate Ca^{2+}

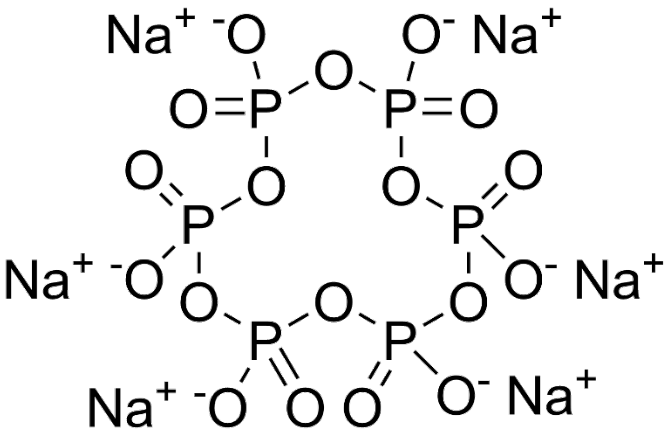
(c) Calgon contains metal which is 2^{nd} most abundant in earth's crust

(d) Calgon is polymeric and water soluble

Answer: (c)

Solution:

* Calgon (Sodium hexametaphosphate) is also known as Graham's salt. It has a polymeric chain structure and is water soluble



* When added to hard water, the following reaction takes place

$Na_6P_6O_{18} \rightarrow 2Na^+ + Na_4P_6O_{18}^{2-}$

Calgon

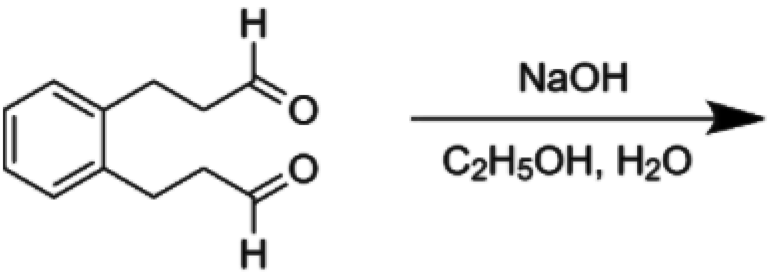
$M^{2+} + Na_6P_6O_{18}^{2-} \rightarrow [Na_2MP_6O_{18}]^{2-} + 2Na^+$

(M = Mg, Ca)

The complex ion keeps the Mg^{2+} and Ca^{2+} ion in the solution and not precipitated

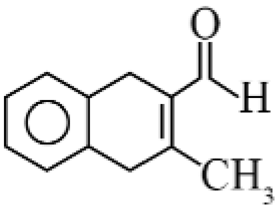
* Second most abundant metal in earth's crust is iron and is not present in Calgon

Question: Final product of the reaction is

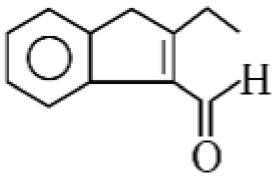


Options:

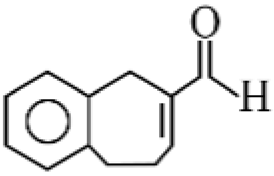
(a)



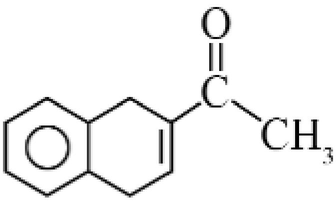
(b)



(c)

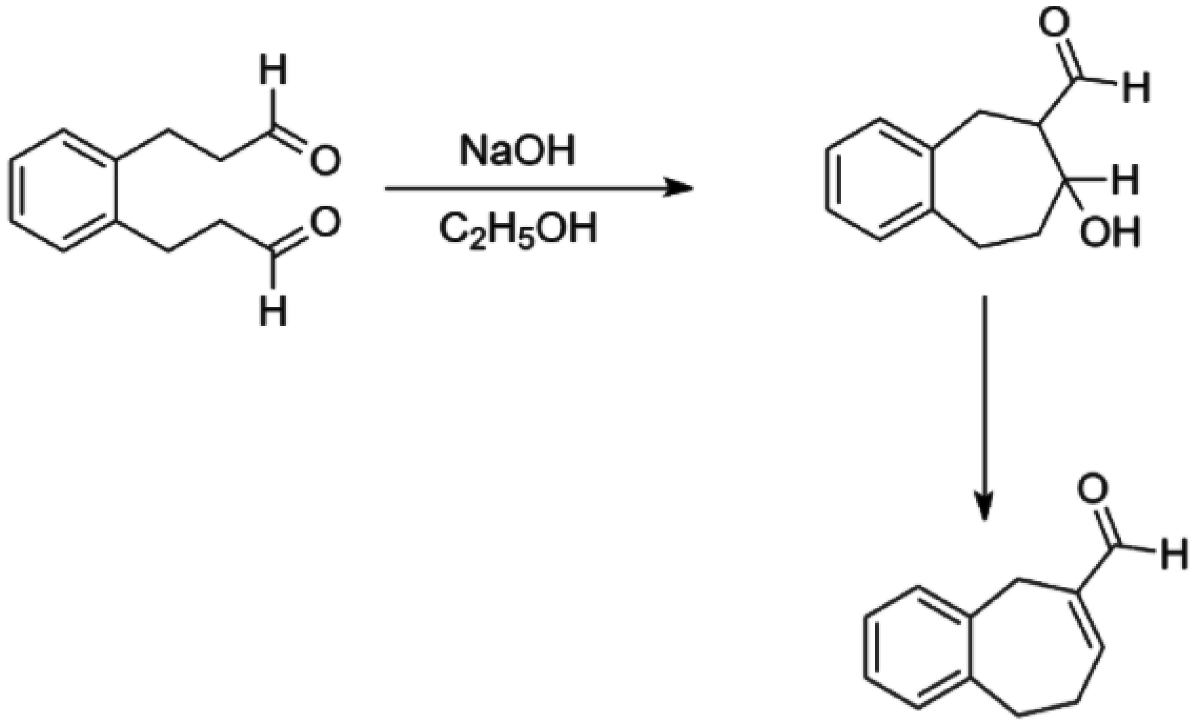


(d)



Answer: (c)

Solution:



Question: 2,4 DNP test is given by:

Options:

(a) Aldehyde

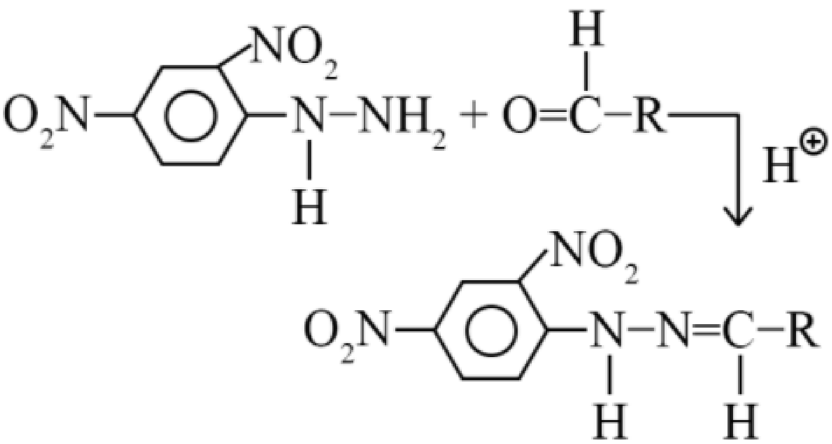
(b) Amine

(c) Ester

(d) Halogens

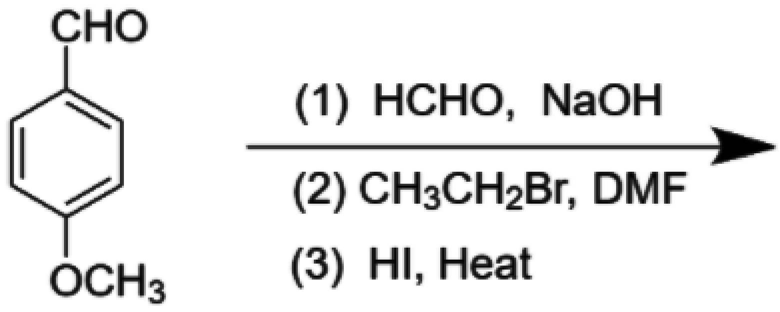
Answer: (a)

Solution:



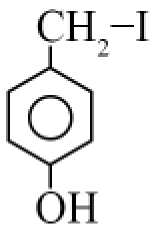
Both aldehyde and ketones gives the 2,4 DNP test

Question:

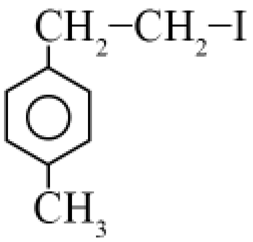


Options:

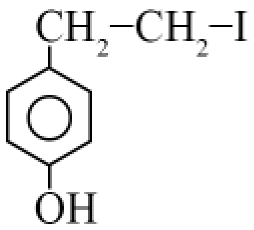
(a)



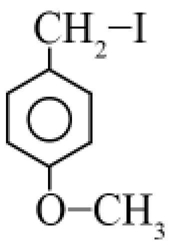
(b)



(c)

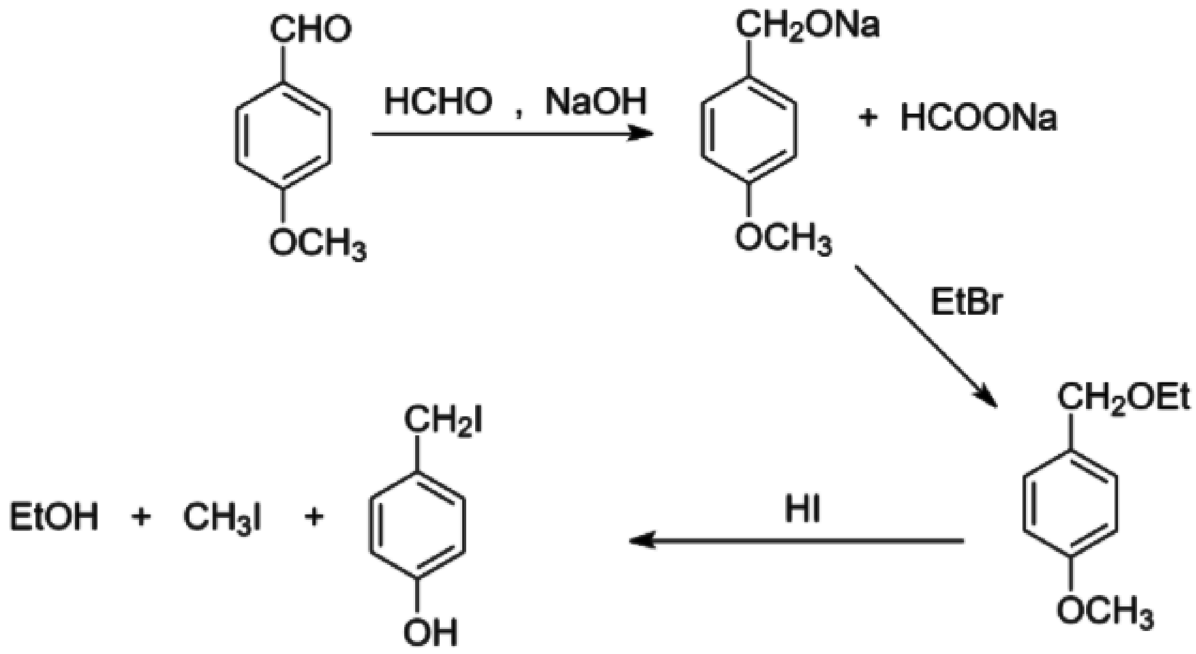


(d)



Answer: (a)

Solution:



Question: Match the following:

Column-I	**Column-II**
(A) Sodium carbonate	**(P)** Deacon
(B) Titanium	**(Q)** Castner-kellner
(C) Chlorine	**(R)** Van-arkel
(D) Sodium Hydroxide	**(S)** Solvay

Options:

(a) $A \to S$; $B \to R$; $C \to Q$; $D \to P$

(b) $A \to Q$; $B \to R$; $C \to P$; $D \to S$

(c) $A \to S$; $B \to R$; $C \to P$; $D \to Q$

(d) $A \to Q$; $B \to P$; $C \to R$; $D \to S$

Answer: (c)

Solution:

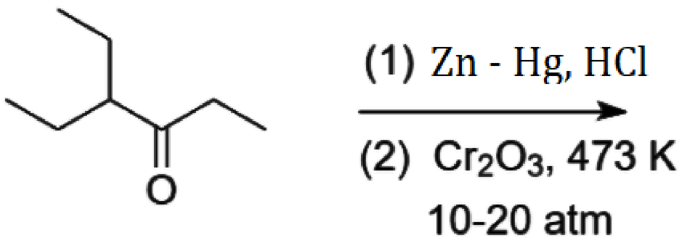
Sodium carbonate is manufactured by Solvay process.

Titanium is refined by Van-Arkle method.

Chlorine is manufactured by Deacon's process.

Sodium hydroxide is manufactured by Castner-kellner process.

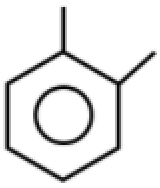
Question:



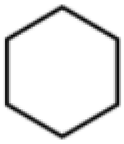
Options:

(a)

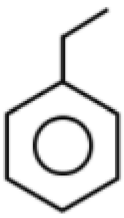
(b)



(c)

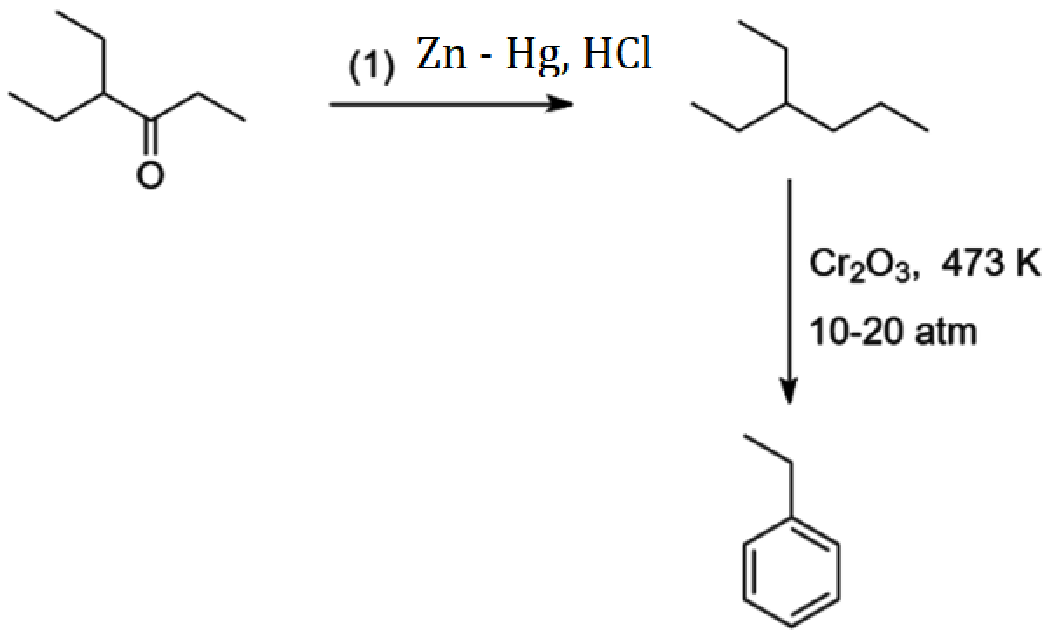


(d)



Answer: (d)

Solution:



Question: Match the following.

Column-I	**Column-II**
(A) Sucrose	**(P)** α-D glucose and β-D fructose
(B) Lactose	**(Q)** β -D galactose and β-D glucose
(C) Maltose	**(R)** α-D glucose and α-D glucose
(D) Cellulose	**(S)** β-D glucose and β-D glucose

Options:

(a) $A \rightarrow P; B \rightarrow Q; C \rightarrow R; D \rightarrow S$

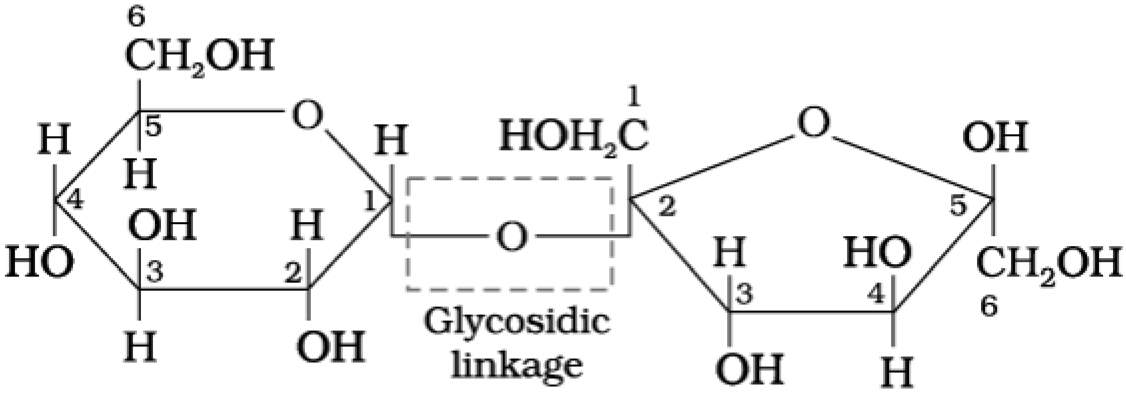
(b) $A \rightarrow Q; B \rightarrow R; C \rightarrow S; D \rightarrow P$

(c) $A \rightarrow R; B \rightarrow S; C \rightarrow P; D \rightarrow Q$

(d) $A \rightarrow S; B \rightarrow P; C \rightarrow Q; D \rightarrow R$

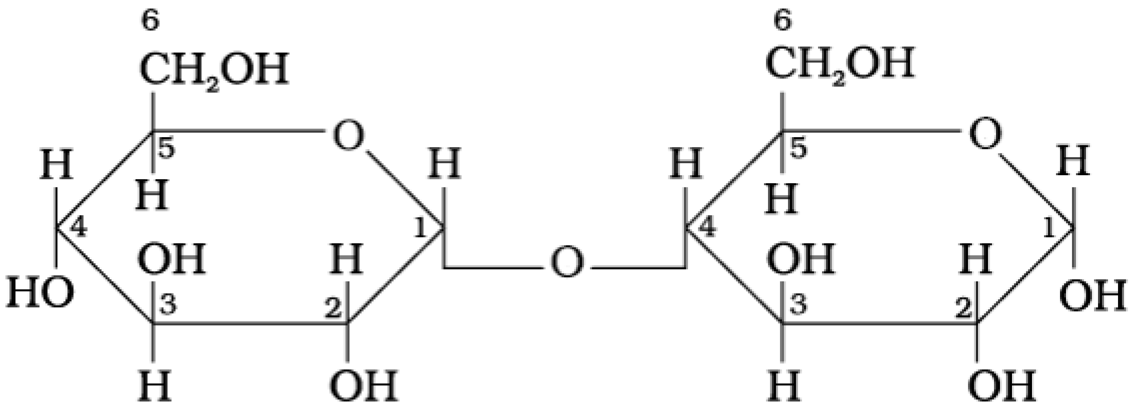
Answer: (a)

Solution:



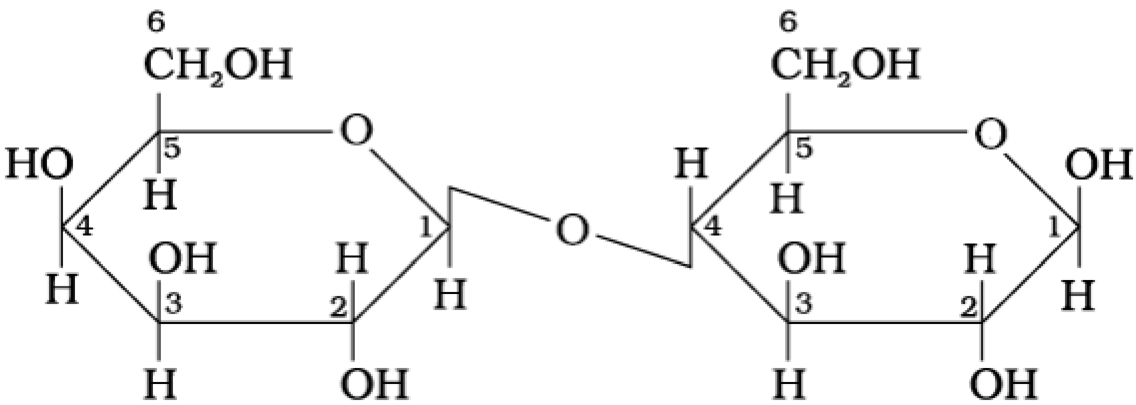
α – D – Glucose β – D – Fructose

Sucrose



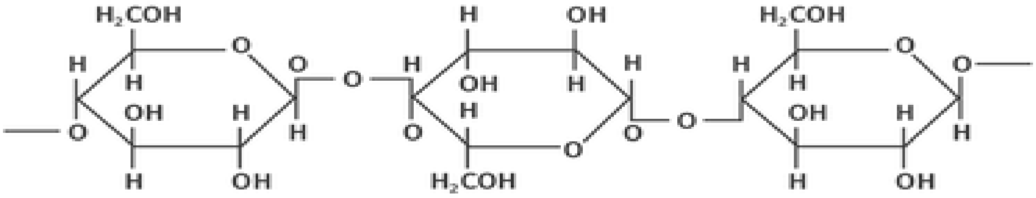
(I) α – D – Glucose (II) α – D – Glucose

Maltose



β – D – Galactose β – D – Glucose

Lactose



Question: Seliwanoff and Xanthoproteic test are respectively used for the identification of:
Options:

(a) Proteins, Ketoses

(b) Ketoses, Proteins

(c) Aldoses, Ketoses

(d) Ketoses, Aldoses

Answer: (b)

Solution:

1) Seliwanoff test is for carbohydrate. It distinguishes between aldoses and ketose sugar

2) Xanthoproteic test is for protein

Question: What is the ratio of number of octahedral voids per unit cell in HCP/CCP?

Answer: 1.50

Solution: The number of octahedral voids is equal to effective number of atoms in both HCP and CCP structures

Thus,

number of octahedral voids in HCP = 6

number of octahedral voids in CCP = 4

Ratio $= \frac{6}{4} = 1.5$

JEE-Main-26-02-2021-Shift-2 (Memory Based)

MATHEMATICS

Question: $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$.

Find the value of $a + b - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) - \left(\frac{a^4 + b^4}{5}\right) \ldots$

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$

$\tan^{-1}\left[\frac{a+b}{1-ab}\right] = \frac{\pi}{4}$

At $b = 1 - ab$

$(1+a)(1+b) = 2$

Now, $(a+b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) \ldots$

$= \left(a - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} + \ldots\right) + \left(b - \frac{b^2}{2} + \frac{b^3}{3} - \frac{b^4}{4} + \ldots\right)$

$= \log(1+a) + \log(1+b)$

$= \log(1+a)(1+b)$

$= \log 2$

Question: 3, 3, 4, 4, 4, 5, 5 find the probability for 7 digit number such that number is divisible by 2

Options:

(a) $\frac{1}{7}$

(b) $\frac{3}{7}$

(c) $\frac{4}{7}$

(d) $\frac{6}{7}$

Answer: (b)
Solution:

Numbers given are 3, 3, 4, 4, 4, 5, 5

Total number of 7 digit number is $\frac{7!}{2!.3!.2!} = 210$

Number divisible by '2' has '4' at unit place

$\therefore$ Total favourable case = $\frac{1 \times 66}{2!.2!.2!} = 90$

$\therefore$ Required probability = $\frac{90}{210} = \frac{3}{7}$

Question: Mirror image of (1, 3, 5) w.r.t plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma) = ?$

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

Equation of line perpendicular to plane $4x - 5y + 2z = 8$ and passing through (1, 3, 5) is

$$\frac{x-1}{4} = \frac{y-3}{-5} = \frac{z-5}{2} = \lambda$$

Any general point on this line is $P(4\lambda + 1, 3 - 5\lambda, 2\lambda + 5)$

Let P lies on plane,

$\therefore 4(4\lambda + 1) - 5(-5\lambda + 3) + 2(2\lambda + 5) = 8$

$45\lambda = 9$

$\Rightarrow \lambda = \frac{1}{5}$

$\therefore P = \left(\frac{9}{5}, 2, \frac{27}{5}\right)$

As P is mid point of (1, 3, 5) and (α, β, γ)

$\therefore \alpha = \frac{13}{5}, \beta = 1, \gamma = \frac{29}{5}$

$$\alpha+\beta+\gamma=\frac{47}{5}$$
$$5(\alpha+\beta+\gamma)=47$$

Question: $f(x)$ is differentiable function at $x=a$, such that $f'(a)=2, f(a)=4$. Find $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$$\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$$

On applying L-Hospital's Rule

$$\lim_{x\to a}\frac{f(a)-af'(x)}{1}=f(a)-af'(a)=4-2a$$

Question: The locus of mid point of the line segment from (3, 2) to the circle $x^2+y^2=1$ which touch the circle ay point P is a circle with radius r. what is the value of r

Options:

(a) 1

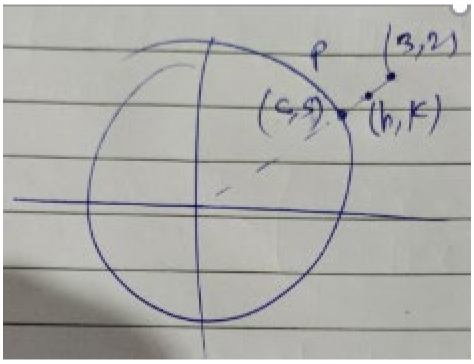
(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

Answer: (b)

Solution:



$\dfrac{c+3}{2},\dfrac{2+5}{2}$

$h=\dfrac{c+3}{2},\ k=\dfrac{2+5}{2}$

$2h-3=c,\ 2k-2=5$

$4h^2+9-12h+4k^2+4-8k=1$

$h^2+k^2+\dfrac{9}{4}-3h-2k+1-\dfrac{1}{4}=0$

$r=\sqrt{9+1-\dfrac{9}{4}-1+\dfrac{1}{4}}$

$=\dfrac{1}{2}$

Question: The slope of the tangent to curve is $\dfrac{xy^2+y}{x}$ and it intersects the line $x+2y=4$ at $x=-2$, then the value of 'y' is (3, y) lies on the curve?

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$\dfrac{dy}{dx}=\dfrac{xy^2+y}{x}\Rightarrow\dfrac{dy}{dx}-\dfrac{y}{x}=y^2$

$\dfrac{1}{y^2}\dfrac{dy}{dx}-\dfrac{1}{yx}=1$

Put $-\dfrac{1}{y}=t\Rightarrow\dfrac{1}{y^2}\dfrac{dy}{dx}=\dfrac{dt}{dx}$

$\Rightarrow\dfrac{dt}{dx}+\dfrac{t}{x}=1$

$I.F.=e^{\int\frac{dx}{x}}=\ln x=x$

$$\therefore t(x)=\int x\,dx=\frac{x^2}{2}+C$$

$$\Rightarrow \frac{-x}{y}=\frac{x^2}{2}+C \text{ (passes through (3, y))}$$

$$\Rightarrow -\frac{3}{y}=\frac{9}{2}+C \Rightarrow -\frac{3}{y}-\frac{9}{2}$$

$$\Rightarrow -\frac{x}{y}=\frac{x^2}{2}-\frac{9}{2}-\frac{3}{y}$$

$$\Rightarrow \frac{3-x}{y}=\frac{x^2-9}{2}$$

$$\Rightarrow y=\frac{2(3-x)}{x^2-9}$$

At $x=-2$; $y=\frac{2\times 5}{(-5)}=-2$

Question: $f(x)=\int_1^x \frac{\ln(1+t)}{t}dt,\ f(e)+f\left(\frac{1}{e}\right)=$

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$$f(x)=\int_1^x \frac{\ln(1+t)\,dt}{t} \quad : \quad f\left(\frac{1}{x}\right)=\int_1^{\frac{1}{x}} \frac{\ln(1+t)}{t}dt$$

Let $t=\frac{1}{u} \Rightarrow dt=-\frac{1}{u}du$

$$\therefore f\left(\frac{1}{x}\right)=\int_1^x \frac{\ln\left(1+\frac{1}{u}\right)}{\left(\frac{1}{u}\right)}\left(-\frac{1}{u}\right)du=-\int_1^x \frac{\ln\left(\frac{1+u}{u}\right)}{u}du$$

$$\therefore f(x)+f\left(\frac{1}{x}\right)=\int_1^x\left[\frac{\ln(1+t)}{t}-\frac{\ln(1+t)}{t}+\frac{\ln t}{t}\right]dt$$

$$f(x)+f\left(\frac{1}{x}\right)=\int_1^x \frac{\ln t}{t}dt=\left[\frac{(\ln t)^2}{2}\right]_1^x=\frac{1}{2}\ln^2 x$$

$\therefore f(e)+f\left(\frac{1}{e}\right)=\frac{1}{2}\ln^2 e=\frac{1}{2}$

Question: $f(x)=\int_1^x e^t f(t)\,dt+e^x$; $f(x)$ is a differentiable function $x\in R$. Then $f(x)=$

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$f(x)=\int_1^x e^t f(t)\,dt+e^x$

$f'(x)=e^x f(x)+e^x$

$\frac{dy}{dx}=e^x(y+1)$

$\int\frac{dy}{(y+1)}=\int e^x\,dx$

$\log|y+1|=e^x+C$

$y+1=\pm e^C e^{e^x}$

$y+1=k.e^{e^x} \qquad \left(\text{Put } \pm e^C=k\right)$

At $x=1, y=0$

$\Rightarrow 1=ke^e$

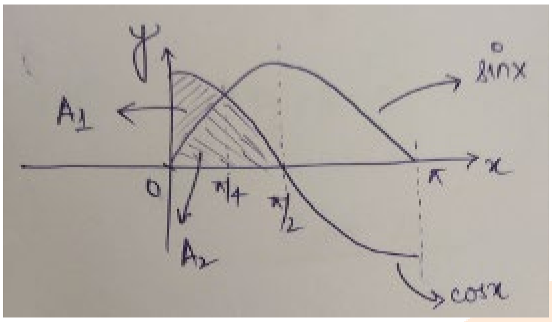
$k=\frac{1}{e^e}$

$\Rightarrow y+1=\frac{e^{e^x}}{e^e}$

$\Rightarrow f(x)=\frac{e^{e^x}-1}{e^e}$

Question: If A_1 is area between the curves $y = \sin x$, $y = \cos x$ and y-axis for $0 \le x \le \frac{\pi}{2}$ and A_2 is area between $y = \sin x$ and $y = \cos x$ and x-axis for $0 \le x \le \frac{\pi}{2}$, the find $\frac{A_2}{A_1}$

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:



$$A_1 = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\, dx = (\sin x + \cos x)_0^{\frac{\pi}{4}} = \sqrt{2} - 1$$

$$A_2 = \int_0^{\frac{\pi}{4}} \sin x\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\, dx = (-\cos x)_0^{\frac{\pi}{4}} + (\sin x)_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left(-\frac{1}{\sqrt{2}} + 1\right) + \left(1 - \frac{1}{\sqrt{2}}\right) = 2 - \sqrt{2}$$

$$\frac{A_2}{A_1} = \frac{2 - \sqrt{2}}{\sqrt{2} - 1} = \sqrt{2}$$

Question: $P_n = \alpha^n + \beta^n$, $\alpha + \beta = 1$, $\alpha\beta = -1$, $P_{n-1} = 11$, $P_{(n+1)} = 29$, then $(P_n)^2$

Answer: 324.00
Solution:

$$(\alpha + \beta) P_n = (\alpha + \beta)(\alpha^n + \beta^n)$$

$$(\alpha + \beta) P_n = \alpha^{n+1} + \beta^{n+1} + \alpha\beta(\alpha^{n-1} + \beta^{n-1})$$

$$\Rightarrow (1) P_n = P_{n+1} - P_{n-1} = 29 - 11 = 18$$

$$\Rightarrow P_n^2 = 324$$

Question: Let $A(1, 4)$ and $B(1, -5)$ be two points let P be the point on $(x-1)^2+(y-1)^2=1$. Find maximum value of $(PA)^2+(PB)^2$.

Answer: 53.00

Solution:

Let $P(1+\cos\theta, 1+\sin\theta)$

$\therefore PA^2+PB^2=\cos^2\theta+(\sin\theta-3)^2+\cos^2\theta+(\sin\theta+6)^2$

$=2\cos^2\theta+2\sin^2\theta+6\sin\theta+45$

$=47+6\sin\theta$

So, it will be maximum when $\sin\theta=1$

$\therefore \left(PA^2+PB^2\right)_{\max}=47+6=53$

Question: Let L be a line of intersection of $x+2y+z=0$ and $y+z=4$. If $P(\alpha, \beta, \gamma)$ is foot of perpendicular from $(3, 2, 1)$ on L. Find $21(\alpha+\beta+\gamma)$.

Options:

(a)

(b)

(c)

(d)

Answer: 98.00

Solution:

D.R. of line L:

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}=(1,-1,1)$$

Put z = 0 in both planes $\Rightarrow y=4,\ x=-2$

$\therefore$ Equation of line L $\Rightarrow \dfrac{x+2}{1}=\dfrac{y+4}{-1}=\dfrac{z-0}{1}=\lambda$

Let point P on line is $(\lambda-2, 4-\lambda, \lambda)$ & A(3, 2, 1)

$\because AP \perp$ line

$\therefore (1)(\lambda-5)+(-1)(2-\lambda)+(1)(\lambda-1)=0$

$\Rightarrow 3\lambda=8 \Rightarrow \lambda=\dfrac{8}{3}$

$\Rightarrow P\left(\dfrac{2}{3}, \dfrac{4}{3}, \dfrac{8}{3}\right)$

$\Rightarrow 21(\alpha+\beta+\gamma)=21\left(\dfrac{2}{3}+\dfrac{4}{3}+\dfrac{8}{3}\right)$

$$= 21\left(\frac{14}{3}\right) = 98$$

Question: How many four digit numbers are there where g.c.d. with 18 is '3'.
Options:
(a)
(b)
(c)
(d)
Answer: 1000.00
Solution:
Number of required numbers
= 4 digit numbers divisible by 3 – 4 digit numbers divisible by 6 – 4 digit numbers divisible by 9 + 4digit numbers divisible by 18
= 3000 – 1500 – 1000 + 200
= 1000

Question: The prime factorization of a number 'n' is given as $n = 2^x \times 3^y \times 5^z$, $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$. Find out the odd divisors of n including 1
Answer: 12.00
Solution:

$y + z = 5$: $\frac{1}{y} + \frac{1}{z} = \frac{5}{6}$ $\Rightarrow (y, z) = (2, 3)$ or (3, 2)

$\therefore$ Number of odd divisors of n = $2^x \cdot 3^y \cdot 5^z$ is $(y+1)(z+1)$

$= 3 \times 4 = 12$

Question: -16, 8, -4, 2, ….., A.M and G.M of p^{th} and q^{th} terms are roots of $4x^2 - 9x + 5 = 0$ then $p + q =$
Answer: 10.00
Solution:
Given sequence is -16, 8, -4, 2

It is a GP with common ratio $r = -\frac{1}{2}$

Its nth term is $a_n = (-16)\left(-\frac{1}{2}\right)^{n-1}$

Roots of $4x^2 - 9x + 5 = 0$ are $1, \frac{5}{4}$

$\therefore GM \le AM \Rightarrow \therefore GM = 1$

Now GM of pth and qth term $= \sqrt{(-16)\left(-\frac{1}{2}\right)^{p-1}.(-16)\left(-\frac{1}{2}\right)^{q-1}}$

$\Rightarrow 16\left(-\frac{1}{2}\right)^{\frac{p+q-2}{2}} = 1$

$\Rightarrow \left(-\frac{1}{2}\right)^{\frac{p+q-2}{2}} = \frac{1}{16}$

$\Rightarrow \frac{p+q-2}{2} = 4$

$\Rightarrow p+q = 10$

Question: The value of square of slope of the common tangent to the curves $4x^2+9y^2=36$ and $(2x)^2+(2y)^2=31$

Options:

(a)

(b)

(c)

(d)

Answer: 3.00

Solution:

Given ellipse are $\frac{x^2}{9}+\frac{y^2}{4}=1$ and $\frac{x^2}{\left(\frac{31}{4}\right)}+\frac{y^2}{\left(\frac{31}{4}\right)}=1$

Let equation of common tangent to ellipse with slope 'm' is

$y = mx+\sqrt{9m^2+4}$ and $y = mx+\sqrt{\frac{31}{4}m^2+\frac{31}{4}}$

$\therefore\ 9m^2+4 = \frac{31}{4}m^2+\frac{31}{4}$

$\Rightarrow \frac{5m^2}{4} = \frac{15}{4}$

$\Rightarrow m^2 = 3$

Question: $\sum_{n=1}^{18}(x_i-\alpha)=36; \sum_{n=1}^{18}(x_i-\beta)^2=90$ and the standard deviation is

Find $|\beta-\alpha|$

Answer: 0.00

Solution:

Let $\alpha=\beta$

$\because$ Standard deviation remains unchanged if observations are added or subtracted by a fixed number

$$\therefore S.D.=\sqrt{\frac{\sum_{i=1}^{18}(x_i-\alpha)^2}{18}-\left[\frac{\sum_{i=1}^{18}(x_i-\alpha)}{18}\right]^2}$$

$$=\sqrt{\left(\frac{90}{18}\right)-\left(\frac{36}{18}\right)^2}=1\text{, which is given}$$

Hence, $\alpha=\beta$ according to the conditions

$\therefore |\beta-\alpha|=0$