JEE-Main-24-02-2021-Shift-2

PHYSICS

Question: Two electrons are fixed at a separation of 2d from each other. A proton is placed at the midpoint and displaced slightly in a direction perpendicular to line joining the two electrons. Find the frequency of oscillation of proton.

Options:

(a) $f = \frac{1}{2\pi}\sqrt{\frac{2ke^2}{md^3}}$

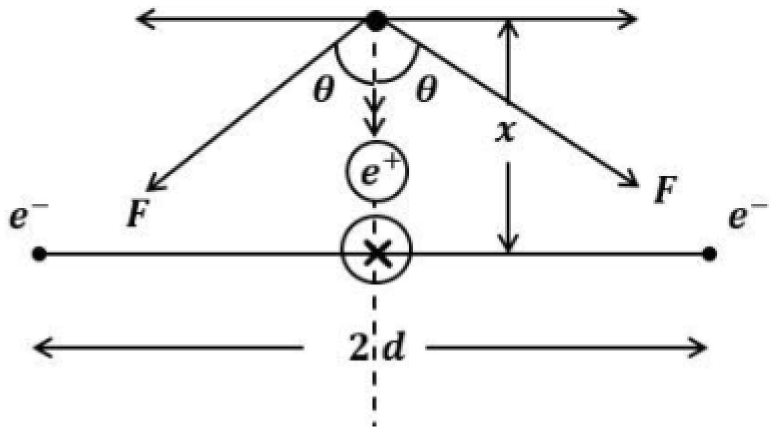
(b) $f = \frac{1}{2\pi}\sqrt{\frac{ke^2}{md^3}}$

(c) $f = \frac{1}{2\pi}\sqrt{\frac{ke^2}{2md^3}}$

(d) None of these

Answer: (a)

Solution:



$$F\cos\theta.2 = m\omega^2 x$$

$$\Rightarrow \frac{k\,e.e}{\left(d^2+x^2\right)}\cdot\frac{2x}{\sqrt{d^2+x^2}} = m\omega^2 x$$

$$\Rightarrow \frac{2ke^2 x}{d^3} = m\omega^2 x \qquad \text{(taking x << d]}$$

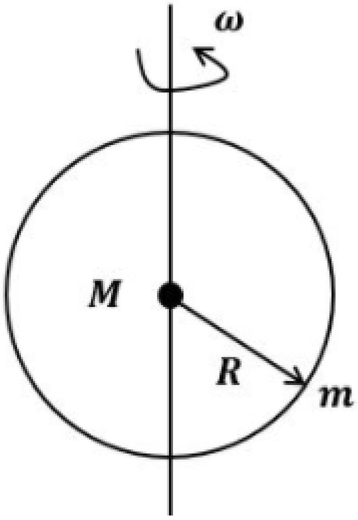
On solving-

$$f = \frac{1}{2\pi}\sqrt{\frac{2ke^2}{md^3}}$$

Question: The weight of a person on pole is 48 kg then the weight on equator is? Give [R = 6400 km]

Options:

(a) 48
(b) 48.83
(c) 47.84
(d) 47
Answer: (c)
Solution:

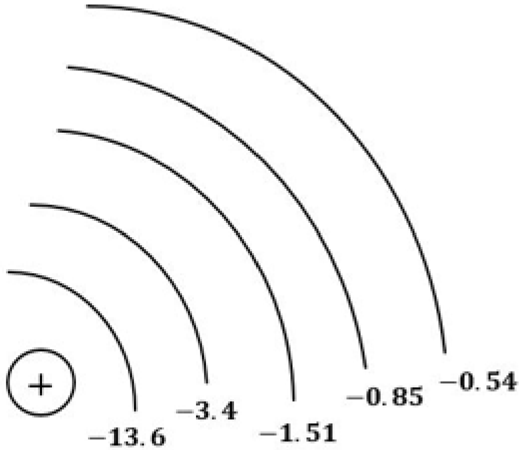


At pole
$$\frac{GMm}{R^2} = 48\,kg \;...(i)$$
At equator
$$\frac{GMm}{R^2} - mR\omega^2 = x \;...(ii)$$
Dividing eq. (ii) by eq. (i)
$$1 - \frac{\omega^2 R^3}{GM} = \frac{x}{48}$$
On putting all the values in this eqn.
$x = 47.83\,kg.$

Question: Two bodies A & B have masses 1 kg & 2 kg respectively have equal momentum. Find the ratio of kinetic energy?
Options:
(a) 1: 1
(b) 2: 1
(c) 1: 4
(d) 1: 2
Answer: (b)
Solution:
$$K = \frac{P^2}{2m}$$
$$\frac{K_A}{K_B} = \frac{m_B}{m_A}$$ [As momentum is same for both]

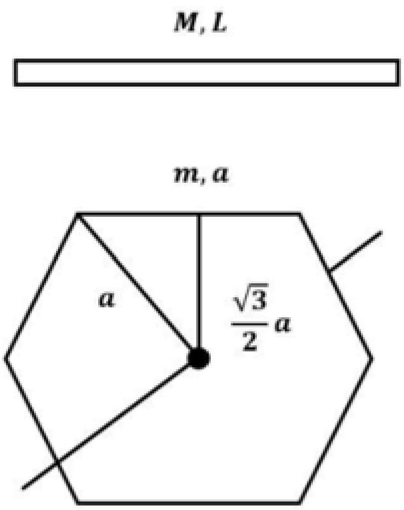
$= \frac{2}{1}$

Question: Which transition in hydrogen spectrum has the maxima frequency?
Options:
(a) $3 \to 2$
(b) $5 \to 4$
(c) $9 \to 5$
(d) $2 \to 1$
Answer: (d)
Solution:



As n increases, difference between n^{th} and $(n+1)^{th}$ orbit energy decreases.
So as per given options $2 \to 1$ transition will have maximum energy & hence maximum frequency.

Question: A rod of mass M, length L is bent in the form of hexagon. Then MOI about axis passing through geometric centre & perpendicular to plane of body will be ?
Options:
(a) $6ML^2$
(b) $\frac{ML^2}{6}$
(c) $\frac{ML^2}{2}$
(d) $\frac{5ML^2}{216}$
Answer: (d)
Solution:



$m = \dfrac{M}{6}$ and $a = \dfrac{L}{6}$

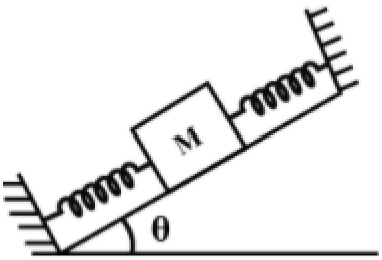
$$I = 6\left[\frac{ma^2}{16} + m\left[\frac{\sqrt{3}a}{2}\right]^2\right]$$

$$= 6\,ma^2\left[\frac{1}{12} + \frac{3}{4}\right]$$

$$= 6 \times \frac{M}{6} \times \frac{L^2}{36} \times \frac{5}{6}$$

$$= \frac{5ML^2}{216}$$

Question: Find the time period of SHM of the block of mass M.



Options:

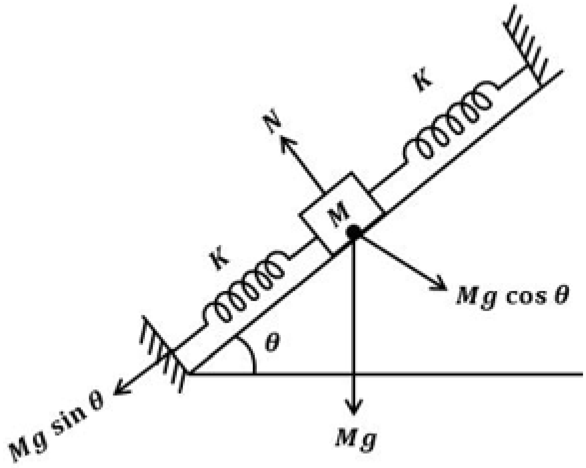
(a) $T = 2\pi\sqrt{\dfrac{M}{2K}}$

(b) $T = 2\pi\sqrt{\dfrac{M}{K}}$

(c) $T = 2\pi\sqrt{\dfrac{2M}{K}}$

(d) $T = 2\pi\sqrt{\dfrac{M}{4K}}$

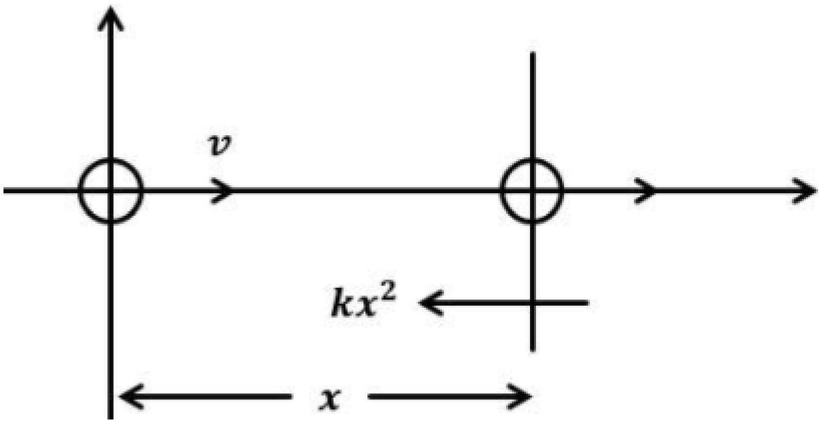
Answer: (a)
Solution:



Constant force doesn't change ω of the system. (Constant force means force that has constant magnitude and direction. In the direction of oscillation these forces have constant contribution.)
So, due to parallel combination of springs-
$K_{eq} = 2K$

Therefore, $\omega = \sqrt{\frac{K_{eq}}{M}} \Rightarrow T = 2\pi\sqrt{\frac{M}{2K}}$

Question: A particle is projected on x axis with velocity v. A force is acting on it in opposite direction , which is proportional to the square of its position. At what distance from origin the particle will stop. [mass is m and constant of proportionality → k]
Options:
(a) $\sqrt[3]{\frac{mv_0^2}{k}}$
(b) $\sqrt[3]{\frac{3mv_0^2}{k}}$
(c) $\sqrt[3]{\frac{3}{2}\frac{mv_0^2}{k}}$
(d) $\sqrt[3]{\frac{mv_0^2}{2k}}$
Answer: (c)
Solution:



Let particle will stop at l distance

$$\frac{-kx}{m}=\frac{vdv}{dx}$$

$$\int_0^l -\frac{kx^2}{m}dx=\int_v^0 vdv$$

$$\frac{-k}{m}\frac{l^3}{3}=\frac{-v^2}{2}$$

$$\Rightarrow l=\sqrt[3]{\frac{3mv_0^2}{2k}}$$

Question: Two cars are approaching each other each moving with a speed v. Find the beat frequency as heard by driver of one car both are emitting sound of frequency f_0 .

Options:

(a) Beat frequency $=\frac{2vf_0}{C-v}$

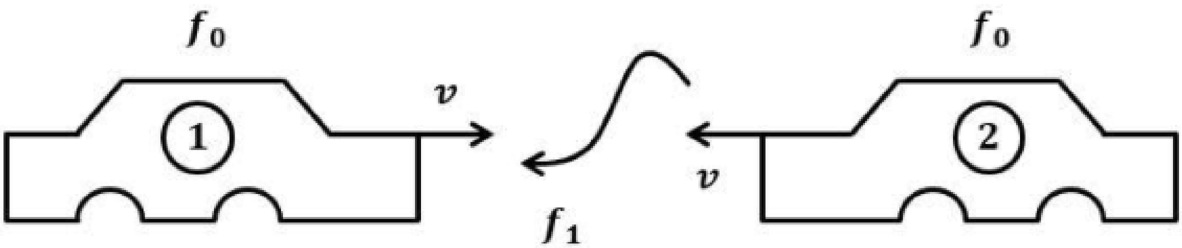
(b) Beat frequency $=\frac{2vf_0}{C+v}$

(c) Beat frequency $=\frac{vf_0}{C-v}$

(d) None of these

Answer: (a)

Solution:



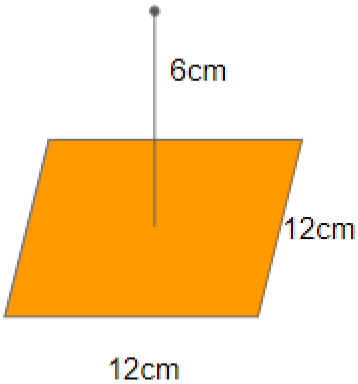
$f_B=f_1-f_2$

[Where f_B is beat frequency]

$$= f_0\left(\frac{C+v}{C-v}\right) - f_0$$

$$\Rightarrow f_B = \frac{2vf_0}{C-v}$$

Question: Find the flux of point charge ‘q’ through the square surface ABCD as shown.



Options:

(a) $\frac{q}{6 \in_0}$

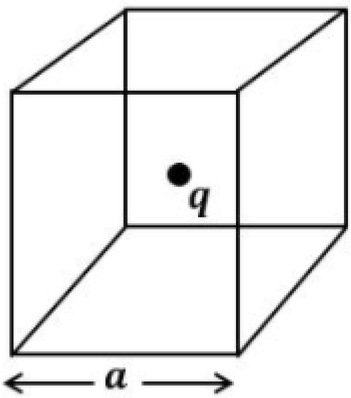
(b) $\frac{q}{\in_0}$

(c) $\frac{q}{4 \in_0}$

(d) $\frac{q}{2 \in_0}$

Answer: (a)

Solution:

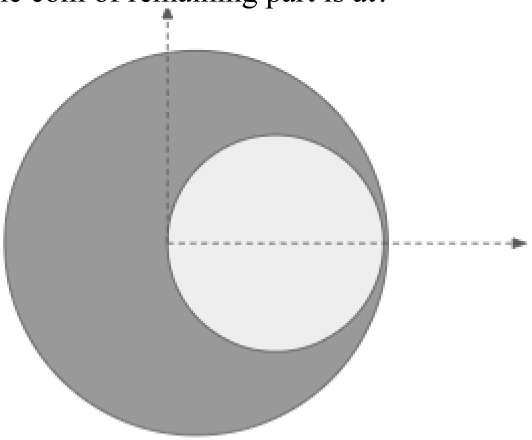


Lets assume a cube of slide a and charge is at it’s centre.

So, from whole cube flux coming out $= \frac{q}{\in_0}$

So, flux coming out from one surface $= \dfrac{q}{6\in_0}$

Question: If a solid cavity whose diameter is removed from a solid sphere of radius R, then the com of remaining part is at?



Options:

(a) $x = \dfrac{-R}{3}$

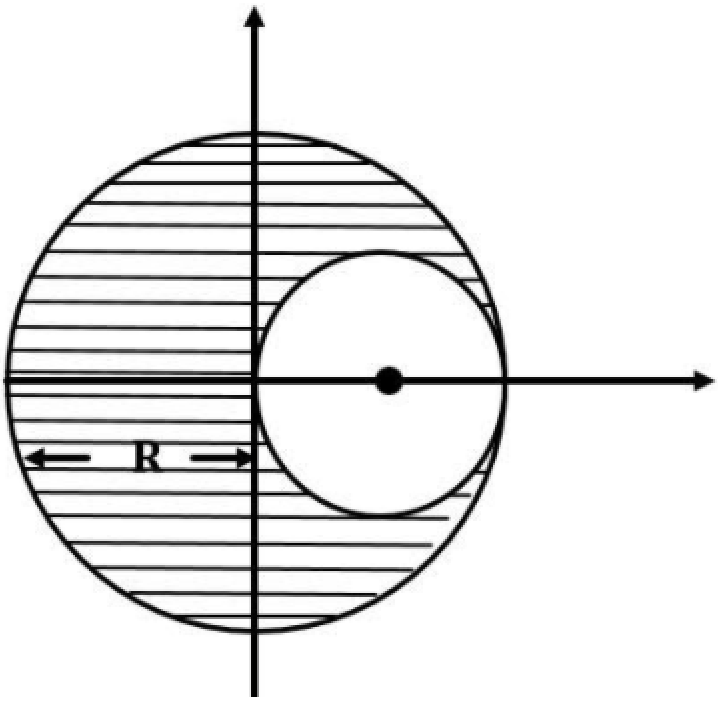
(b) $x = \dfrac{-R}{7}$

(c) $x = \dfrac{-R}{6}$

(d) $x = \dfrac{-R}{14}$

Answer: (c)

Solution:



$$M = \sigma\pi R^2, m = -\sigma\pi\left(\frac{R}{2}\right)^2$$

$$x_{cm} = \frac{M(0) + m\left(\frac{R}{2}\right)}{M + m} = \frac{\left(\frac{m}{M}\right)\left(\frac{R}{2}\right)}{1 + \frac{m}{M}}$$

$$\Rightarrow x_{cm} = \frac{-R}{6}$$

Question: In a YDSE experiment, if Red light is replaced by violet light then the fringe width will be

Options:

(a) decrease

(b) increase

(c) may increase or decrease

(d) None of these

Answer: (a)

Solution:

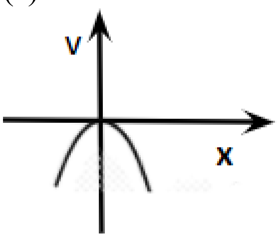
$$\beta = \frac{\lambda D}{d}$$

As decreases for violet light, the fringe width will also decrease.

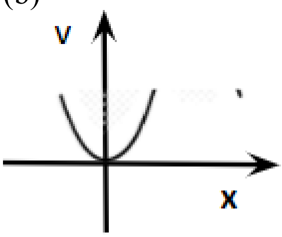
Question: The graph of V versus x in an SHM is (v : velocity, x : displacement)

Options:

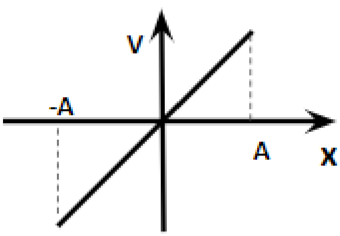
(a)



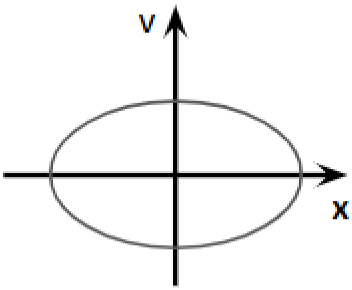
(b)



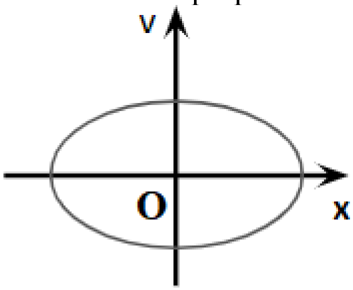
(c)



(d)

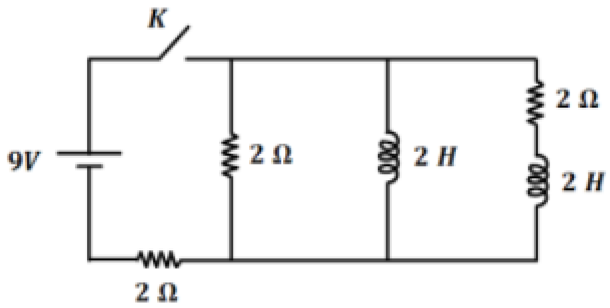


Answer: (d)
Solution:
A simple harmonic motion is an example of periodic motion. In simple harmonic motion, a particle is accelerated towards a fixed point (in this case, O) and the acceleration of the particle will be proportional to the magnitude of the displacement of the particle.



Question: If the de Broglie wavelengths of an alpha particle and a proton are the same, then the ratio of their velocities is:
Options:
(a) $\frac{1}{4}$
(b) $\frac{4}{1}$
(c) $\frac{1}{2}$
(d) 1
Answer: (a)
Solution:
$$\frac{h}{m_\alpha v_\alpha} = \frac{h}{m_p v_p}$$
$$\Rightarrow \frac{v_\alpha}{v_p} = \frac{m_p}{m_\alpha} = \frac{1}{4}$$

Question: Find the current through the battery just after the key is closed.



Options:

(a) $\frac{9}{4}A$

(b) $\frac{9}{2}A$

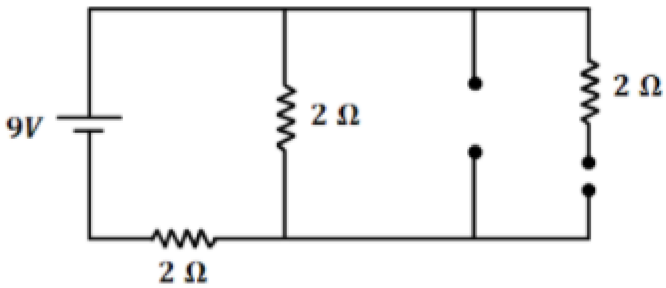
(c) $\frac{9}{1}A$

(d) None of these

Answer: (a)

Solution:

Just after the key is closed, circuit will be



So current in the circuit

$$I = \frac{9}{R_{eq}} = \frac{9}{4} Amp$$

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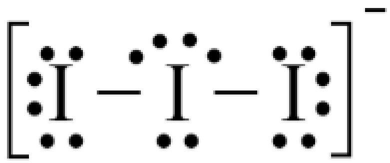
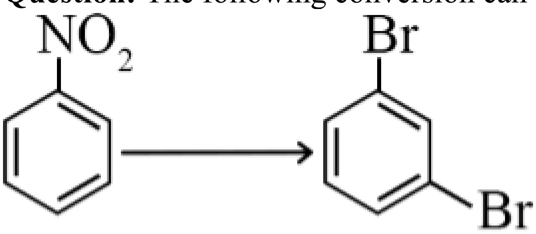
CHEMISTRY

Question: S in BUNA-S stands for?
Options:
(a) Styrene
(b) Strength
(c) Stoichiometry
(d) Secondary
Answer: (a)
Solution: BUNA-S ⇒ Styrene butadiene

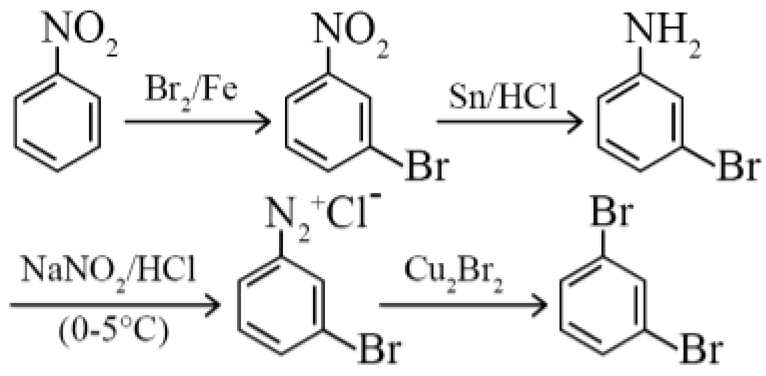
Question: Bond angle and shape of I_3^- ion is?
Options:
(a) 180° and sp^3d
(b) 180° and sp^3d^2
(c) 90° and sp^3d
(d) 90° and sp^3d^2
Answer: (a)
Solution:



Question: The following conversion can take place by:

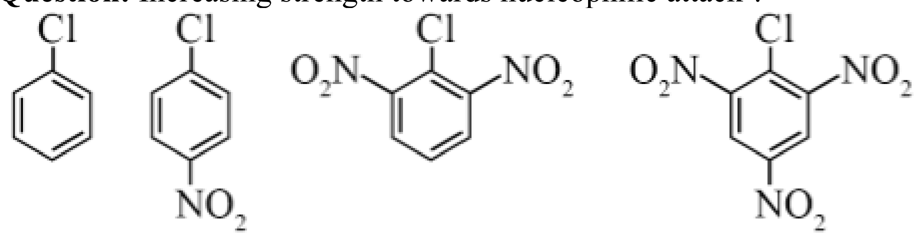


Options:
(a) (i) Br_2/Fe (ii) Sn/HCl
(b) (i) Br_2/Fe (ii) Sn/HCl (iii) $NaNO_2$/HCl (iv) Cu_2Br_2
(c) (i) Cu_2Br_2 (ii) Sn/HCl (iii) Br_2/Fe
(d) None of these
Answer: (b)
Solution:



Question: According to Bohr's model which of the following transition will be having maximum frequency?
Options:
(a) 3 to 2
(b) 5 to 4
(c) 4 to 3
(d) 2 to 1
Answer: (d)
Solution: 2 to 1 (Lyman series)
Lyman series falls in UV region. Therefore higher energy than other radiations.

Question: PbI_2 given as = 0.1 M ; $K_{sp} = 8\times10^{-9}$
Find solubility of Pb^{2+}
Options:
(a) 1.4×10^{-3}
(b) 2×10^{-4}
(c) 1.26×10^{-3}
(d) 1.8×10^{-2}
Answer: (c)
Solution:
$Ksp = 4s^3$
$8\times10^{-9} = 4s^3$
$s^3 = 2\times10^{-9}$
$s = 1.26\times10^{-3}$

Question: Increasing strength towards nucleophilic attack ?


(I) (II) (III) (IV)

Options:
(a) (I) < (II) < (III) < (IV)
(b) (IV) < (III) < (II) < (I)

(c) (IV) < (II) < (III) < (I)
(d) (I) < (III) < (II) < (IV)
Answer: (a)
Solution: Electron withdrawing groups increases the rate of nucleophilic substitution reaction, due to increase of electrophilic character of carbon involved in C – X bond.

Question:
Statement 1: Hydrogen is most abundant in universe but not so in Earth's troposphere.
Statement 2: Hydrogen is the lightest element.
Options:
(a) Statement 1 is correct and Statement 2 is incorrect.
(b) Statement 1 is incorrect and Statement 2 is correct.
(c) Statement 1 is correct and Statement 2 is correct explanation for statement 1.
(d) Statement 1 is correct and Statement 2 is incorrect explanation for statement 1.
Answer: (c)
Solution: Due to light weight of hydrogen, it is not abundant in earth's troposphere.

Question: Which of the following salts help in blood clotting?
Options:
(a) $FeCl_3$
(b) $Mg(HCO_3)_2$
(c) $NaHCO_3$
(d) $FeSO_4$
Answer: (a)
Solution: Blood being a colloidal solution its coagulation can be understood by Hardy-Schulz's law which states that higher is the charge on cation, higher will be its efficiency to coagulate the colloidal solution.
In the present case, ferric chloride has Fe^{3+}. Hence, ferric chloride is more effective in enhancing the coagulation rate of blood and stop the bleeding from the cut.

Question: Match the following:

A	B
(p) Al	(i) Siderite
(q) Zn	(ii) Malachite
(r) Fe	(iii) Calamine
(s) Cu	(iv) Bauxite

Options:
(a) p → (iv); q → (iii), r → (i), s → (ii)
(b) p → (i); q → (ii), r → (iv), s → (iii)
(c) p → (iv); q → (iii), r → (ii), s → (i)
(d) p → (iii); q → (iv), r → (i), s → (ii)
Answer: (a)
Solution:
Siderite - $FeCO_3$
Malachite – $CuCO_3$ $Cu(OH)_2$
Calamine – $ZnCO_3$
Bauxite – Al_2O_3

Question: What will be the magnetic moments (spin only values) of the following complexes?

$\left[FeCl_4\right]^{2-}, \left[Co\left(C_2O_4\right)_3\right]^{3-}, MnO_4^{2-}$

Options:

(a) $\sqrt{3}, 0, 0$

(b) $\sqrt{24}, 0, \sqrt{3}$

(c) $\sqrt{24}, \sqrt{24}, 0$

(d) $\sqrt{3}, 0, \sqrt{24}$

Answer: (b)

Solution:

$[FeCl_4]^{2-} \Rightarrow Fe^{2+} \Rightarrow 3d^6$

$\mu = \sqrt{24}\ B.M$

$\left[Co\left(C_2O_4\right)_3\right]^{3-} \Rightarrow Co^{3+} \Rightarrow 3d^6$

$\mu = 0\ B.M$

$MnO_4^{2-} \Rightarrow Mn^{6+} \Rightarrow 3d^1$

$\mu = \sqrt{3}\ B.M$

Question: Compare the wavelength in flame test for LiCl, NaCl, KCl, RbCl, CsCl

Options:

(a) NaCl < CsCl < LiCl < RbCl < KCl

(b) CsCl < NaCl < LiCl < KCl < RbCl

(c) RbCl < KCl < LiCl < CsCl < NaCl

(d) CsCl < NaCl < KCl < LiCl < RbCl

Answer: (b)

Solution:

Compound	**Wavelength (λ) (in nm)**
LiCl	670.8
NaCl	584.2
KCl	766.5
RbCl	780
CsCl	455

Question: Choose incorrect statement:

Options:

(a) RuO_4 is oxidizing agent

(b) OsO_4 is reducing agent

(c) Cr_2O_3 is amphoteric

(d) Red colour of ruby is due to Co^{3+}

Answer: (b)

Solution: $OsO_4 \Rightarrow$ Maximum oxidation state (+8) Hence, it can get reduce and oxidise other species i.e. it is a oxidizing agent.

Question: Which of the following has highest M.P.?

Options:

(a) MgO

(b) LiF

(c) NaCl
(d) LiCl
Answer: (a)
Solution:
$MgO \Rightarrow Mg^{2+}, O^{2-}$
Due to higher charge, ionic character will be high and hence, melting point also.

Question: Arrange the following in the increasing order of their density: Zn, Fe, Cr, Co
Options:
(a) Zn < Cr < Co < Fe
(b) Fe < Co < Cr < Zn
(c) Fe < Cr < Co < Zn
(d) Zn < Cr < Fe < Co
Answer: (d)
Solution:
$\text{Density } = \dfrac{\text{mass}}{\text{volume}}$
firstly, metallic radius of transition elements (first transition series) decreases from Sc to Ni then increases from Ni to Zn.

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MATHEMATICS

Question: Given, $f(0)=1, f(2)=e^2, f'(x)=f'(2-x)$, then the value of $\int_0^2 f(x)dx$ is

Options:
(a) $1-e^2$
(b) $1+e^2$
(c) $3e$
(d) e^2
Answer: (b)
Solution:

$f'(x)=f'(2-x)$

Integrate w.r.t. x

$f(x)=-f(2-x)+C$

Put $x=0$

$f(0)=-f(2)+C$

$1=-e^2+C$

$C=1+e^2$

$\therefore f(x)=-f(2-x)+1+e^2$

$\Rightarrow f(x)+f(2-x)=1+e^2 \quad ...(\text{i})$

Let, $I=\int_0^2 f(x)dx \quad ....(\text{ii})$

$I=\int_0^2 f(2-x)dx \quad ...(\text{iii})$

$(\text{ii})+(\text{iii})$

$2I=\int_0^2 [f(x)+f(2-x)]dx$

$2I=\int_0^2 (1+e^2)dx \quad (\text{from (i)})$

$2I=2(1+e^2)$

$\Rightarrow I = 1 + e^2$

Question: The area of region defined by $5x^2 \le y \le 2x^2 + 9$

Options:

(a) $6\sqrt{3}$

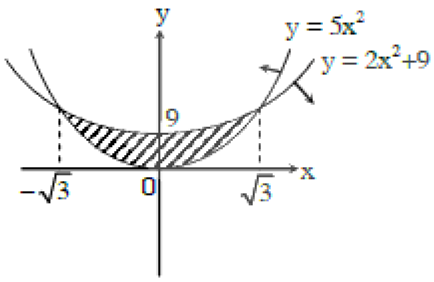
(b) $12\sqrt{3}$

(c) $18\sqrt{3}$

(d) $9\sqrt{3}$

Answer: (b)

Solution:



Intersection points

$5x^2 = 2x^2 + 9$

$3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$

Area = $\int_{-\sqrt{3}}^{\sqrt{3}} \left(2x^2 + 9 - 5x^2\right) dx$

$= 2\int_{0}^{\sqrt{3}} \left(9 - 3x^2\right) dx$

$= 2\left[9x - x^3\right]_0^{\sqrt{3}}$

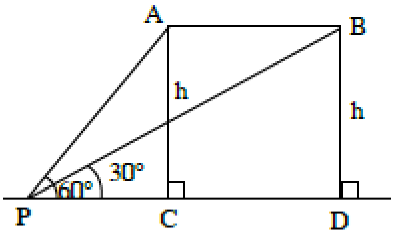
$= 2\left(9\sqrt{3} - 3\sqrt{3}\right)$

$= 12\sqrt{3}$

Question: A plane is flying horizontally with speed $120\, m/s$. Its angle of elevation from a point on ground is $60°$. After 20s angle of elevation is $30°$. Find height of plane

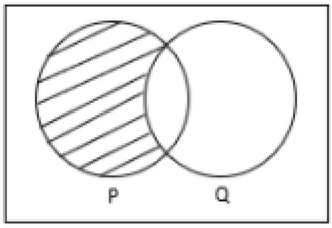
Options:

(a) $1200\sqrt{3}$
(b) $2400\sqrt{3}$
(c) $600\sqrt{3}$
(d) $1500\sqrt{3}$
Answer: (a)
Solution:



$$AB = h\cot 60° = \frac{h}{\sqrt{3}}$$
$$AC = h\cot 30° = \sqrt{3}h$$
$$BC = AC - AB = \sqrt{3}h - \frac{h}{\sqrt{3}} = 120\times 20$$
$$\Rightarrow \frac{2h}{\sqrt{3}} = 120\times 20$$
$$\Rightarrow h = 1200\sqrt{3}m$$

Question: Negation of the statement $\sim p \vee (p \wedge q)$ is
Options:
(a) $p \wedge \sim q$
(b) $p \vee \sim q$
(c) $\sim p \wedge q$
(d) $\sim p \vee \sim q$
Answer: (b)
Solution:
$\sim(\sim p \vee (p \wedge q)) = p \wedge \sim (p \wedge q)$
= only p



Question: Vertices of Δ are (a, c), (2, b) and (a, b), a, b, c are in A.P. centroid is $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are roots of $ax^2 + bx + 1 = 0$ then $\alpha^2 + \beta^2 - \alpha\beta =$
Options:

(a) $-\dfrac{71}{256}$
(b) $\dfrac{71}{256}$
(c) $\dfrac{69}{256}$
(d) $-\dfrac{69}{256}$
Answer: (a)
Solution:
a, b, c are in A.P. $\Rightarrow 2b = a + c \qquad ...(\text{i})$

Centroid $= \left(\dfrac{2a+2}{3}, \dfrac{2b+c}{3}\right) = \left(\dfrac{10}{3}, \dfrac{7}{3}\right)$

$\Rightarrow \dfrac{2a+2}{3} = \dfrac{10}{3} \Rightarrow a = 4$

and $\dfrac{2b+c}{3} = \dfrac{7}{3} \Rightarrow a + c + c = 7$ (from (i))

$\Rightarrow 2c + 4 = 7 \Rightarrow c = \dfrac{3}{2}$

Put in (i)

$2b = 4 + \dfrac{3}{2} = \dfrac{11}{2}$

$b = \dfrac{11}{4}$

α, β are root of $ax^2 + bx + 1 = 0$

$\Rightarrow \alpha + \beta = \dfrac{-b}{a} = \dfrac{-11}{16}, \alpha\beta = \dfrac{1}{a} = \dfrac{1}{4}$

So, $\alpha^2 + \beta^2 - \alpha\beta = (\alpha + \beta)^2 - 3\alpha\beta$

$= \left(\dfrac{-11}{16}\right)^2 - \dfrac{3}{4}$

$= \dfrac{121 - 192}{256} = \dfrac{-71}{256}$

Question: If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0, f(x) = 1, f'(0) = 2, f''(x) \neq 0$ then $f(1)$ lies in

Options:
(a) (0, 3)
(b) (6, 9)
(c) [9, 12]
(d) [5, 7]
Answer: (b)
Solution:
$f(x) f''(x) - [f'(x)]^2 = 0$

$\Rightarrow \frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$

$\Rightarrow \int \frac{f''(x)}{f'(x)} dx = \int \frac{f'(x)}{f(x)} dx$

$\Rightarrow \log f'(x) = \log f(x) + \log c$

$\Rightarrow f'(x) = c\, f(x)$

Put $x = 0$

$f'(0) = c\, f(0)$

$\Rightarrow 2 = c$

$\Rightarrow f'(x) = 2f(x)$

$\Rightarrow \int \frac{f'(x)}{f(x)} dx = \int 2\, dx$

$\log f(x) = 2x + \mathrm{D}$

$f(x) = e^{\mathrm{D}} e^{2x}$

$f(x) = \mathrm{K}\, e^{2x} \qquad \left(\text{Put } e^{\mathrm{D}} = k\right)$

Put $x = 0$

$f(0) = \mathrm{K}$

$\Rightarrow \mathrm{K} = 1$

$\Rightarrow f(x) = e^{2x}$

$\Rightarrow f(1) = e^{2}$

which lies in $(6, 9)$

Question: The value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

Options:

(a) $\frac{1}{\sqrt{7}}$

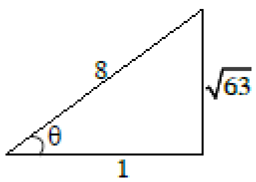
(b) $\frac{1}{\sqrt{5}}$

(c) $\frac{2}{\sqrt{3}}$

(d) none of these

Answer: (a)

Solution:



Let $\sin^{-1}\frac{\sqrt{63}}{8}=\theta$

$\Rightarrow \sin\theta=\frac{\sqrt{63}}{8}$

$\Rightarrow \cos\theta=\frac{1}{8}$

$\Rightarrow \cos\frac{\theta}{2}=\sqrt{\frac{1+\cos\theta}{2}}=\sqrt{\frac{1+\frac{1}{8}}{2}}=\sqrt{\frac{9}{16}}=\frac{3}{4}$

So, $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)=\tan\left(\frac{\theta}{4}\right)$

$=\sqrt{\frac{1-\cos\left(\frac{\theta}{2}\right)}{1+\cos\left(\frac{\theta}{2}\right)}}=\sqrt{\frac{1-\frac{3}{4}}{1+\frac{3}{4}}}=\frac{1}{\sqrt{7}}$

Question: The number of natural numbers less than 7,000 which can be formed by using the digits $0, 1, 3, 7, 9$ (repetition of digits allowed) is equal to

Options:

(a) 250

(b) 374

(c) 372

(d) 375

Answer: (b)

Solution:

7 and 9 cannot occur at first place

Hence, required number of natural numbers less than 7000

$=3\times5\times5\times5-1=375-1=374$

(we have subtracted 1 for the case 0000 case)

Question: Find the value of ${}^{n+1}C_2+2\left({}^2C_2+{}^3C_2+...+{}^nC_2\right)=?$

Options:

(a) $\frac{n(n+1)(2n-1)}{6}$

(b) $\frac{n(n+1)(2n+1)}{6}$

(c) $\frac{(n-1)n(n+1)}{6}$

(d) $\frac{n(n+1)}{2}$

Answer: (b)

Solution:

$S = {}^2C_2 + {}^3C_2 + \ldots + {}^nC_2 = {}^{n+1}C_3$

$\therefore\ {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$= \frac{(n+1)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$

$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)n(n-1)}{6} = \frac{n(n+1)}{6}(2n+1)$

Question: If A and B are subsets of X = {1, 2, 3, 4, 5} then find the probability such that $n(A \cap B) = 2$

Options:

(a) $\frac{65}{2^7}$

(b) $\frac{65}{2^9}$

(c) $\frac{35}{2^9}$

(d) $\frac{135}{2^9}$

Answer: (d)

Solution:

Required probability = $\frac{{}^5C_2 \times 3^3}{4^5}$

$= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$

Question: A curve $y = f(x)$ passing through the point (1, 2) satisfies the differential equation $x\frac{dy}{dx} + y = bx^4$ such that $\int_1^2 f(y)\,dy = \frac{62}{5}$. The value of b is

Options:

(a) 10

(b) 11

(c) $\frac{32}{5}$

(d) $\frac{62}{5}$

Answer: (a)

Solution:

$$\frac{dy}{dx}+\frac{y}{x}=6x^3$$

I.F. $= e^{\int \frac{dy}{dx}} = x$

$$\therefore\ yx=\int bx^4dx=\frac{bx^5}{5}+C$$

Passes through (1, 2), we get

$$2=\frac{b}{5}+C \qquad ....(\text{i})$$

Also, $\int_1^2\left(\frac{bx^4}{5}+\frac{C}{x}\right)dx=\frac{65}{2}$

$$\Rightarrow \frac{b}{25}\times 32+C\ln 2-\frac{b}{25}=\frac{62}{5}$$

$$\Rightarrow C=0 \ \ \& \ \ b=10$$

Question: A curve $y=ax^2+bx+c$ passing through the point (1, 2) has slope at origin equal to 1, then ordered triplet (a, b, c) may be

Options:

(a) (1, 1, 0)

(b) $\left(\frac{1}{2},1,0\right)$

(c) $\left(-\frac{1}{2},1,1\right)$

(d) $\left(2,-1,0\right)$

Answer: (a)

Solution:

$$2=a+b+c \qquad ....(\text{i})$$

$$\frac{dy}{dx}=2ax+b \Rightarrow \left.\frac{dy}{dx}\right|_{(0,0)}=1$$

$$\Rightarrow b=1 \Rightarrow a+c=1$$

Question: The value of $\int_1^3\left[x^2-2x-2\right]dx$ ([.] denotes greatest integer function)

Options:

(a) -4

(b) -5

(c) $-1-\sqrt{2}-\sqrt{3}$

(d) $1-\sqrt{2}-\sqrt{3}$

Answer: (c)

Solution:

$$I=\int_1^3 -3\,dx+\int_1^3\left[(x-1)^2\right]dx$$

Put $x-1=t;\ dx=dt$

$$I=(-6)+\int_0^2\left[t^2\right]dt$$

$$I=(-6)+\int_0^1 0\,dt+\int_1^{\sqrt{2}}1\,dt+\int_{\sqrt{2}}^{\sqrt{3}}2\,dt+\int_{\sqrt{3}}^{2}3\,dt$$

$$I=-6+\left(\sqrt{2}-1\right)+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}$$

$$I=-1-\sqrt{2}-\sqrt{3}$$

Question: Which of the following conic has tangent '$x+\sqrt{3}y-2\sqrt{3}$' at point $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$?

Options:

(a) $x^2+9y^2=9$

(b) $y^2=\frac{x}{6\sqrt{3}}$

(c) $x^2-9y^2=10$

(d) $x^2=\frac{y}{6\sqrt{3}}$

Answer: (a)

Solution:

Tangent to $x^2+9y^2=a$ at point $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$ is $x\left(\frac{3\sqrt{3}}{2}\right)+9y\left(\frac{1}{2}\right)=9$

Option (a) is true

Question: Equation of plane passing through $(1,\,0,\,2)$ and line of intersection of planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)=-2$ is

Options:

(a) $\vec{r}\cdot\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$

(b) $\vec{r}\cdot\left(3\hat{i}+10\hat{j}+3\hat{k}\right)=7$

(c) $\vec{r}\cdot\left(\hat{i}+\hat{j}-3\hat{k}\right)=4$

(d) $\vec{r}\cdot\left(\hat{i}+4\hat{j}-\hat{k}\right)=-7$

Answer: (a)

Solution:
Plane passing through intersection of plane is
$\left\{\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=-1\right\}+\lambda\left\{\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)+2\right\}=0$

Passing through $\hat{i}+2\hat{k}$, we get

$(3-1)+\lambda(\lambda+2)=0 \Rightarrow \lambda=-\dfrac{2}{3}$

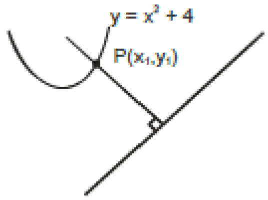
Hence, equation of plane is $3\left\{\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right\}-2\left\{\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)+2\right\}=0$

$\Rightarrow \vec{r}\cdot\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$

Question: A is 3×3 square matrix and B is 3×3 skew symmetric matrix and X is a 3×1 matrix, then equation $\left(A^2B^2-B^2A^2\right)X=0$ (Where O is a null matrix) has/have

Options:
(a) Infinite solution
(b) No solution
(c) Exactly one solution
(d) Exactly two solution
Answer: (a)
Solution:
$A^T=A$, $B^T=-B$
Let $A^2B^2-B^2A^2=P$
$P^T=\left(A^2B^2-B^2A^2\right)^T=\left(A^2B^2\right)^T-\left(B^2A^2\right)^T$
$=\left(B^2\right)^T\left(A^2\right)^T-\left(A^2\right)^T\left(B^2\right)^T$
$=B^2A^2-A^2B^2$
$\Rightarrow$ P is skew-symmetric matrix
$\Rightarrow |P|=0$
Hence $PX=0$ have infinite solution

Question: Find a point on the curve $y=x^2+4$ which is at shortest distance from the line $y=4x-1$.
Options:
(a) (2, 8)
(b) (1, 5)
(c) (3, 13)
(d) (-1, 5)
Answer: (a)
Solution:
$\left.\dfrac{dy}{dx}\right|_p=4$
$\therefore\ 2x_1=4$



$\Rightarrow x_1 = 2$

$\therefore$ Point will be (2, 8)

Question: Let $f(x)=\begin{cases} -55x & ; \quad x<-5 \\ 2x^3-3x^2-120x & ; \quad -5\le x<4 \\ 2x^3-3x^2-36x+10 & ; \quad x\ge 4 \end{cases}$

Then interval in which f(x) is monotonically increasing is

Options:

(a) $(-5,-4)\cup(4,\infty)$

(b) $(-\infty,-4)\cup(5,\infty)$

(c) $(-5,4)\cup(5,\infty)$

(d) $(-5,-4)\cup(3,\infty)$

Answer: (a)

Solution:

$$f'(x)=\begin{cases} -55 & ; \quad x<-5 \\ 6(x^2-x-20) & ; \quad -5<x<4 \\ 6(x^2-x-6) & ; \quad x>4 \end{cases}$$

$$f'(x)=\begin{cases} -55 & ; \quad x<-5 \\ 6(x-5)(x+4) & ; \quad -5<x<4 \\ 6(x-3)(x+2) & ; \quad x>4 \end{cases}$$

Hence, $f(x)$ is monotonically increasing is $(-5,-4)\cup(4,\infty)$

Question: If variance of ten numbers 1, 1, 1, 1, 1, 1, 1, 1, 1, k; where $k\in N$, is less than or equal to 10 then maximum value of k is.

Answer: 11.00

Solution:

$\text{var}\le 10$

$$\frac{9(1^2)+k^2}{10}-\left(\frac{9+k}{10}\right)^2\le 10$$

$$90+10k^2-81-k^2-18k\le 1000$$

$9k^2 - 18k \le 991$

$9k(k-2) \le 991$

$\therefore k \in N$

$\therefore$ By hit and trial we observe that max. value of k is 11.

Question: If $a+\alpha=1, \beta+b=2$ and $a(f(x))+\alpha\left(f\left(\frac{1}{x}\right)\right)=\frac{\beta}{x}+bx$, then find value of

$$\frac{\left[f(x)+f\left(\frac{1}{x}\right)\right]}{x+\frac{1}{x}}=$$

Answer: 2.00
Solution:

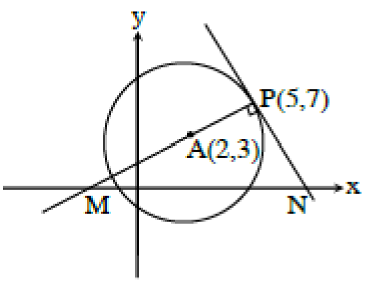
Take $a=\alpha=\frac{1}{2}$ and $b=\beta=1$

Now, $a(f(x))+\alpha\left(f\left(\frac{1}{x}\right)\right)=\beta x+\frac{b}{x}$

$\Rightarrow \frac{1}{2}\left[f(x)+f\left(\frac{1}{x}\right)\right]=x+\frac{1}{x}$

$\Rightarrow \frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=2$

Question: $(x-2)^2+(y-3)^2=25$, Normal and tangent are drawn to it at $(5,7)$. Area of Δ made by normal, tangent and x – axis is A. Find 24A.
Answer: 1225.00
Solution:



Equation of normal at $\mathrm{P}(5,7)$

$y-7=\frac{7-3}{5-2}(x-5)$

$$y-7=\frac{4}{3}(x-5)$$

Put $y=0$

$$-21=4x-20$$
$$4x=-1$$
$$x=\frac{-1}{4}$$
$$\Rightarrow \mathrm{B}\left(\frac{-1}{4},\ 0\right)$$

Equation of tangent at $\mathrm{P}(5,\ 7)$

$$y-7=\frac{-3}{4}(x-5)$$

Put $y=0$

$$-28=-3x+15$$
$$\Rightarrow 3x=43\ \Rightarrow x=\frac{43}{3}$$
$$\Rightarrow \mathrm{C}\left(\frac{43}{3},\ 0\right)\ \Rightarrow \mathrm{BC}=\frac{43}{3}+\frac{1}{4}=\frac{175}{12}$$

So, $24\mathrm{A}=24\times\frac{1}{2}\times\frac{175}{12}\times 7=1225$

Question: Sum of first four terms of $G.P=\frac{65}{12}$. Sum of their reciprocals is $\frac{65}{18}$. Product of first 3 terms is 1. If 3rd term is α, $2\alpha=$

Answer: 3.00

Solution:

Let G.P. is

$$\frac{a}{r},a,ar,ar^2$$

Now, $\frac{a}{r}\cdot a\cdot ar=1\Rightarrow a^3=1\Rightarrow a=1$

Also, $\frac{a}{r}+a+ar+ar^2=\frac{65}{12}$

$$\Rightarrow \frac{1}{r}+1+r+r^2=\frac{65}{12}$$
$$\Rightarrow \frac{1+r+r^2+r^3}{r}=\frac{65}{12} \qquad ...(\text{i})$$

And $\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}=\frac{65}{18}$

$$\Rightarrow r+1+\frac{1}{r}+\frac{1}{r^2}=\frac{65}{18}$$

$$\Rightarrow \frac{r^3+r^2+r+1}{r^2}=\frac{65}{18} \qquad \ldots(\text{ii})$$

$$\frac{(\text{i})}{(\text{ii})} \Rightarrow \frac{r^2}{r}=\frac{18}{12}$$

$$\Rightarrow r=\frac{3}{2}$$

$\therefore$ 3rd term = $\alpha = ar = \frac{3}{2}$

$\therefore \ 2\alpha = 3$

Question: $S_1, S_2, ..., S_{10}$ are 10 students, in how many ways they can be divided in 3 groups A, B and C such that all groups have atleast one student and C has maximum 3 students.
Answer: 31650.00
Solution:
Case 1: C gets exactly 1 student

$$\Rightarrow {}^{10}C_1 \times \left(2^9-2\right)=10\times 510=5100$$

Case 2: C gets exactly 2 students

$$\Rightarrow {}^{10}C_2 \times \left(2^8-2\right)=11430$$

Case 3: C gets exactly 3 students

$$\Rightarrow {}^{10}C_3 \times \left(2^7-2\right)=15120$$

Total number of ways = 5100 + 11430 + 15120 = 31650

Question: $A(5,0)$ and $B(-5,0)$ are two points $PA=3PB$. Then locus of P is a circle with radius ‘r’. Then $4r^2=$
Answer: 525.00
Solution:
Let $P(h,k)$

$$PA=3PB \Rightarrow PA^2=9PB^2$$

$$\Rightarrow (h-5)^2+k^2=9\left[(h+5)^2+k^2\right]$$

$$\Rightarrow h^2+25-10h+k^2=9h^2+225+90h+9k^2$$

$$\Rightarrow 8h^2+8k^2+100h+200=0$$

$$\Rightarrow h^2 + k^2 + \frac{25}{2}h + 25 = 0$$

So, locus is

$$x^2 + y^2 + \frac{25}{2}x + 25 = 0$$

Its radius $= \sqrt{\left(\frac{25}{4}\right)^2 - 25}$

$$\Rightarrow r = \sqrt{\frac{622 - 400}{16}} = \frac{15}{4}$$

$$\Rightarrow 4r^2 = 4\left(\frac{225}{16}\right) = \frac{225}{4}$$