

JEE-Main-25-02-2021-Shift-2 (Memory Based) PHYSICS

Question: Find the flux through shaded region, due to a charge Q placed at a vertex of a cube.



Options:



Answer: (c)

Solution:

If we assume that 8 cubes are placed in such a way that charge Q placed at one corner of the cube is at the centre of those all, then flux passing through one cube will be

$$\frac{Q}{8\varepsilon_0}$$

And now, of the given cube three surfaces are in directly contact with the given charge.

So, flux through these three surfaces will be zero that means $\frac{Q}{8\varepsilon_0}$ flux will pass through remaining 3 surfaces.

So, flux through each one of these surface $=\frac{Q}{24\varepsilon_0}$.

Therefore flux passing through two half of the surfaces will be $= 2 \times \left[\frac{1}{2} \times \frac{Q}{24\varepsilon_0} + \frac{1}{2} \times \frac{Q}{24\varepsilon_0} \right]$

$$=\frac{Q}{24\varepsilon_0}$$



Question: An electron enters with kinetic energy KE_1 between the plates of a capacitor with a velocity making angle ' α ' as shown and leaves with KE_2 making angle ' β '. Find $\frac{KE_1}{KE_2}$?

Options: ²

- (a) $\frac{\cos^2 \alpha}{\cos^2 \beta}$ (b) $\frac{\cos^2 \beta}{\cos^2 \alpha}$
- (c) $\frac{\cos^2 \alpha}{\cos \beta}$
- (c) $\cos \beta$ (d) $\frac{\cos \alpha}{1}$

$\frac{(a)}{\sin \beta}$ **Answer:** (b)

Solution:

Let e^- enters with initial velocity \vec{u} and leaves with final velocity \vec{v} .

 \Rightarrow Horizontal Component of velocity would be same so,

 $u\cos\alpha = v\cos\beta$

$$\Rightarrow \frac{u}{v} = \frac{\cos\beta}{\cos\alpha}$$

So, K.E would be in Ratio

$$\Rightarrow \frac{(K.E)_1}{(K.E)_2} = \frac{\frac{1}{2}m_e u^2}{\frac{1}{2}m_e v^2} = \left(\frac{u}{v}\right)^2 = \left(\frac{\cos\beta}{\cos\alpha}\right)^2$$

Question: If $\overline{A} \times \overline{B} = \overline{B} \times \overline{A}$. Then find the angle between $\overline{A} \& \overline{B}$? **Options:**

(a) Zero
(b) 180°
(c) 360°
(d) All of the above Answer: (d)



Solution:

We have

$$\vec{A} \times \vec{B} = AB\sin\theta \cdot \hat{n}$$

It is given that

 $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

This will be possible only if

$$\left| \vec{A} \times \vec{B} \right| = 0$$

Then $AB\sin\theta = 0$

$$|A| \neq 0 \& |B| \neq 0$$

 $\sin\theta = 0$

 $\theta = n\pi$

Where $n \varepsilon I$

Then $\theta = 0, 180^{\circ}, 360^{\circ}$

Therefore all options are correct. Means (d) option is correct.

Question: Find the time period



Options:

(a) $T = 2\pi \sqrt{\frac{m}{K}}$ (b) $T = 2\pi \sqrt{\frac{m}{4K}}$ (c) $T = 2\pi \sqrt{\frac{m}{2K}}$ (d) $T = 2\pi \sqrt{\frac{m}{8K}}$

Answer: (b) Solution:

Here two springs are connected with mass m in such a way that elongation in one spring will be equal to contraction in other. So,



Both springs are in parallel.

So,
$$K_{eq} = 2K + 2K = 4K$$

Now, $T = 2\pi \sqrt{\frac{m}{K_{eq}}}$
 $\therefore T = 2\pi \sqrt{\frac{m}{4K}}$

Question: Find the dimension of $\frac{e^2}{4\pi\varepsilon_0 hc}$?

Options: (a) $\left[M^0 L^1 T^1\right]$ (b) $\left[M^{0}L^{1}T^{0}\right]$ (c) $\left[M^0 L^0 T^0\right]$ (d) $\left[M^0 L^{-1} T^0 \right]$ Answer: (c)

Solution:

 e^2 $4\pi\varepsilon_0 hc$

- $e \rightarrow$ Electric charge
- $\varepsilon_0 \rightarrow$ Permittivity of free space
- $c \rightarrow$ Speed of light
- $h \rightarrow$ Plank's constant

$$[e] = \left[M^{0}L^{0}T^{1}A^{1} \right]$$
$$[e]^{2} = \left[M^{0}L^{0}T^{2}A^{2} \right]$$
$$[c] = \left[LT^{-1} \right]$$
$$[h] = \left[ML^{2}T^{-1} \right]$$
$$[\varepsilon_{0}] = \left[M^{-1}L^{-3}T^{4}A^{2} \right]$$
$$\left[\frac{e^{2}}{4\pi\varepsilon_{0}hc} \right] = \frac{M^{0}L^{0}T^{0}A^{0}}{\left[M^{-1}L^{-3}T^{4}A^{2} \right] \left[ML^{2}T^{-1} \right] \left[LT^{-1} \right]}$$
$$\left[\frac{e^{2}}{4\pi\varepsilon_{0}hc} \right] = \left[M^{0}L^{0}T^{0}A^{0} \right]$$



Question: A solid spherical ball is rolling without slipping towards a fixed inclined plane with speed V_0 . Find the maximum height, that the ball reached on the inclined.

Rond G≁₀ Fixed **Options:** (a) $h = \frac{7V_0^2}{10g}$ (b) $h = \frac{2V_0^2}{10 g}$ (c) $h = \frac{5}{7} \frac{V_0^2}{g}$ (d) $h = \frac{2}{5} \frac{V_0^2}{g}$ Answer: (a) Solution: $KE_{bottom} = PE_{top}$ $KE_{bottom} = KE_{tr} + KE_{Rrt}$ $=\frac{1}{2}mV_0^2+\frac{1}{2}I\omega^2$ $=\frac{1}{2}mV_0^2+\frac{1}{2}\cdot\frac{2}{5}mR^2\omega^2$ $=\frac{1}{2}mV_{0}^{2}+\frac{1}{5}mr^{2}\left[V_{0}=R\omega\right]$ $=\frac{7}{10}mV_0^2$ $PE_{top} = mgh$ $mgh = \frac{7}{10}mV_0^2$ $h = \frac{7V_0^2}{10g}$



Question: Two small conducting spheres have charges 2.1 nC and - 0.1 nC are touched to each other and then separated by a distance of 0.5 m. Find the force between them? **Options:**

(a) 18 nN (b) 72 nN

(c) 9 nN(d) 36 nN

Answer: (d)

Solution:

Considering both spheres to be identical. So, Net charge would be equally distributed.

Charge on each sphere = 1 nC

$$F = \frac{Kq_1q_2}{r^2} = \frac{(9 \times 10^9)(10^{-9})(10^{-9})}{\left(\frac{1}{2}\right)^2}$$
$$F = 36 \times 10^{-9} N = 36 \, nN$$

Question: Two masses $M_1 \& M_2$ have same kinetic energy. If $V_2 = 2V_1$ then find ratio of their momentum

Options:

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) $\frac{1}{2}$ Answer: (c) Solution:

M₁ & M₂ having speed V₁ & V₂

Such that $V_2 = 2V_1$

--

Now, they are having same $K\varepsilon$.

So,

$$K\varepsilon_{1} = K\varepsilon_{2}$$

$$\frac{1}{2}M_{1}V_{1}^{2} = \frac{1}{2}M_{2}V_{2}^{2}$$

$$M_{1}V_{1}^{2} = M_{2}(2V_{1})^{2}$$

$$M_{1}V_{1}^{2} = M_{2} \times 4.V_{1}^{2}$$

$$\frac{M_{1}}{M_{2}} = 4$$

Using relation $P^2 = 2m(K\varepsilon)$



$$\frac{P_{1}}{P_{2}} = \frac{\sqrt{2M(K\varepsilon_{1})}}{2M_{2}(K\varepsilon_{2})} \qquad \{ \text{We have } K\varepsilon_{1} = K\varepsilon_{2} \& M_{1} = 4M_{2} \}$$

$$\frac{P_{1}}{P_{2}} = \sqrt{4}$$

$$\frac{P_{1}}{P_{2}} = 2$$
Question: Find ratio of $\frac{l_{1}}{l_{2}}$?
$$(a) \frac{1}{2}$$

$$(b) \frac{2}{1}$$

$$(c) \frac{4}{1}$$

$$(d) \frac{1}{4}$$
Answer: (a)
Solution:
$$i_{1} = \frac{2}{6} \times 6 = 2A$$

$$i_{2} = \frac{4}{6} \times 6 = 4A$$

$$\frac{l_{1}}{l_{2}} = \frac{2}{4} = \frac{1}{2}$$

Question: A stone is released from top of a building of height h. When it goes by 5 m from top another stone is dropped from the height which is 25 m below the top of the building. Both the stones reached the ground simultaneously. Find height h of the building.



Options:

(a) 25 m (b) 35 m (c) 40 m (d) 45 m **Answer:** (d) **Solution:**

of first hall take a see to server total hight (h)

Let first ball take n sec to cover total hight (h) then second ball will take (n-1) sec to cover (h-25). Because under free fall stone released from rest cover 5 m is 1s

So for Ist ball

$$h = \frac{1}{2} (10) (n)^2 \dots (i)$$

For IInd ball

$$(h-25) = \frac{1}{2}(10)(n-1)^2$$
 ...(*ii*)

Solving (i) and (ii)

$$\Rightarrow \frac{1}{2}(10)n^2 - 25 = \frac{1}{2}(10)(n-1)^2$$

Solving we get $n = 3 \sec \alpha$

$$h = \frac{1}{2} (10) (3)^2 = 45 \, m$$

Question: Find the truth table of the logic circuit.



Options:

 $\begin{array}{cccc} A & B & Y \\ 0 & 0 & 1 \\ (a) & 0 & 1 & 0 \end{array}$

- 1 0 1 0 1 0
- 1 1 1



Answer: (a) Solution:

We get from circuit diagram.

$$Y = \overline{\left(\overline{AB} + \overline{AB}\right)}$$
$$= \left(\overline{AB}\right)\left(\overline{\overline{AB}}\right)$$
$$= \left(\overline{A} + B\right) \cdot \left(A + \overline{B}\right)$$
$$Y = AB + \overline{AB}$$

Α	В	$Y = AB + \overline{A}\overline{B}$
0	0	1
0	1	0
1	0	0
1	1	1

Question: In a travelling wave, the particle at x = 0, is undergoing SHM given by $y(t) = A\sin(\omega t + \phi_0)$, this particle is at $y = +\frac{A}{2}$ and is moving towards negative y - direction. Find the ϕ_0 ?

Options:

(a)
$$\phi_0 = \frac{\pi}{6}$$

(b) $\phi_0 = \frac{5\pi}{6}$





As per the given condition, (Taking $\phi = 0$ at *MP*), the particle has completed a phase $\frac{5\pi}{6}$

Question: A message signal of frequency f_m is amplitude modulated and sent through a carrier signal of frequency f_c . Find the wavelength of the signal, if C is the speed of light. **Options:**

(a)
$$\frac{C}{f_c}$$

(b) $\frac{C}{f_m}$
(c) $\frac{C}{f_c + f_m}$
(d) $\frac{C}{f_c - F_m}$

Answer: (a) Solution:



In Amplitude modulation, Amplitude of carrier Signal is changed according to message Signal.

So wavelength of modulated signal would be same as carrier signal.

$$f_c \lambda = C \implies \lambda = \frac{C}{f_c}$$

Question: STATEMENT 1 : There are equal degrees of freedom for translational and Rotational motion in a diatomic ideal gas.

STATEMENT 2 : Rotational motion of diatomic molecules follows maxwell distribution. **Options:**

(a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of statement 1.

(b) Statement 1 is true, Statement 2 is true

(c) Statement 1 is false, Statement 2 is false

(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of statement 1.

Answer: (c)

Solution:

Diatomic molecules generally have 5 degree of freedom at low temperature, we can ignore vibrational motion.

So, it have 2-rotational degree of freedom and 3- translational degree of freedom.

And rotational motion of diatomic molecules does not follow maxwell distribution.

So, both statements are wrong.

Question: For a thermodynamic process $PV^{\frac{1}{2}} = \text{constant}$. Find $\frac{T_1}{T_2} = ?$



Options:





(d)
$$\frac{1}{\sqrt{2}}$$

Answer: (c) Solution:

$$PV^{\frac{1}{2}} = \text{constant}$$

$$\Rightarrow \frac{nRT}{V} \left(V^{\frac{1}{2}} \right) = C$$

$$PV = nRT$$

$$P = \frac{nRT}{V}$$

$$T \propto V^{\frac{1}{2}}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{2V_1}{V_1} \right)^{\frac{1}{2}}$$

$$\frac{T_1}{T_2} = \frac{\sqrt{2}}{1}$$



Find x? (h: planck's constant) (use $E = mc^2$) **Options:** (a) x = 10h(b) $x = \frac{h}{10}$ (c) $x = \frac{h}{100}$ (d) x = 100h **Answer:** (a) **Solution:** $\frac{hc}{\lambda} = \left(\frac{x}{3}\right)c^2$ $\Rightarrow x = \frac{3h}{\lambda C} = \frac{3 \times 6.63 \times 10^{-24}}{10 \times 10^{-10} \times 3 \times 10^8}$ x = 10h



JEE-Main-25-02-2021-Shift-2 (Memory Based) CHEMISTRY

Question: Among the first group elements which one is used for photoelectrodes? **Options:**

(a) Cesium

(b) Lithium

(c) Beryllium

(d) Magnesium

Answer: (a)

Solution: The electron in Cesium atoms are stimulated by direct sunlight because I.E., in cesium is very low.

Question: Which purification technique is used for Indium metal? **Options:**

(a) Van arkel method

(b) Liquation

(c) Vapour phase refining

(d) Zone refining

Answer: (d)

Solution: Zone refining is the method based on the principle that the impurities are more soluble in the melt than in the solid state of the metal. This method is very useful for producing semiconductor and other metals of very high purity, e.g., germanium, silicon, boron, gallium and indium.

Question: Composition of German silver? Options: (a) Cu - 50%, Zn - 19%, Ni - 30%(b) Cu - 60%, Zn - 20%, Ni - 20%(c) Cu - 40%, Zn - 48%, Ni - 12%(d) Cu - 80%, Zn - 19%, Ni - 1%Answer: (a) Solution: Cu - 50%, Zn - 19%, Ni - 30%

Question: How will you get 2-bromo-4-nitro ethyl benzene from benzene?

Options:

(a) CH3-CH2-Cl/Anhy AlCl3, Conc. HNO3/H2SO4, Br2/FeBr3

(b) Conc. HNO3/H2SO4, CH3-CH2-Cl/Anhy AlCl3, Br2/FeBr3

(c) CH3-CH2-Cl/Anhy AlCl3, Conc. H2SO4, Br2/FeBr3



(d) None of these

Answer: (a) Solution:



Question: What is the solubility of Ca(OH)₂ in water? (K_{sp} for Ca(OH)₂ = 5.55×10^{-6} **Options:**

(a) 1.11×10^{-2} (b) 1.5×10^{-4} (c) 6.5×10^{-2} (d) 3.4×10^{-3} Answer: (a) Solution: Ca(OH)₂ \Rightarrow Ca⁺² + 2OH⁻ K_{sp} = 4s³ $5.55 \times 10^{-6} = s^{3}$ $s = 1.11 \times 10^{-2}$

Question: Magnetic moment of divalent ion of atomic number = 29 Options: (a) 1.732 B.M (b) 1.44 B.M (c) 2.23 B.M (d) None of these Answer: (a) Solution: Z = 29 is Cu \Rightarrow Cu⁺² = 4s°3d⁹ 11 11 11 11 1 1 1 $\mu = \sqrt{n(n+2)} = \sqrt{1(3)} = \sqrt{3} = 1.73$

Question: Compare bond dissociation energy of F2, Cl2, Br2, I2 :

Options:



(a) $Cl_2 > Br_2 > F_2 > I_2$ (b) $F_2 > Br_2 > Cl_2 > I_2$

(c) $F_2 > Br_2 > I_2 > Cl_2$

(d) $F_2 > Cl_2 > Br_2 > I_2$

Answer: (a) Solution: $Cl_2 > Br_2 > F_2 > I_2$

Question: In which of the following, -COOH group is not present?

Options:

- (a) Benzoic acid
- (b) Sorbic acid
- (c) Sulphonic acid
- (d) Aspirin

Answer: (c) Solution:

NH₂ HO₃S

Sulphonic acid

Question: Arrange the following in the increasing order of their acidic strength?



Options:

(a) (I) < (III) < (IV) < (II)



(b) (III) \leq (II) \leq (IV) \leq (I)

(c) (I) < (II) < (III) < (IV)

(d) (IV) < (III) < (II) < (I)

Answer: (a)

Solution: Acidic strength of phenol is less then carboxylic acid So, (I) is least acidic

Now,



$$\propto \frac{1}{+I \text{ or } + M}$$

Question: Cu exists in FCC lattice having edge length 3. Find density?

Options:

(a) 1.56×10^{-23}

(b) 2.56×10^{-23}

(c) 8.54×10^{-23}

(d) None of these

Answer: (a)
Solution: In Fcc, Z = 4
$$\rho = \frac{Z \times M}{N_a \times a^3} = \frac{4 \times 63.5}{6 \times 10^{23} \times 27}$$
$$= \frac{254}{162 \times 10^{23}} = 1.56 \times 10^{-23}$$

Question: AB electrolyte dissociates 75 %. Find molality if boiling point increases by 4 K

Options:



(a) 4.4 mol/kg

(b) 3.2 mol/kg

(c) 2.4 mol/kg

(d) 5.6 mol/kg

Answer: (a) Solution: $\Delta T_b = 4 \text{ K}$ $K_b = 0.512 \text{ K/kg mol}$ $AB = A^+ + B^ \alpha = \frac{75}{100} = 0.75$ $i = 1 + \alpha (n - 1)$ = 1 + 0.75 (2 - 1) i = 1 + 0.75 i = 1.75 $\Delta T_b = i \times K_b \times m \text{ (for dissociation)}$ $m = \frac{4}{1.75 \times 0.512} = 4.39 \text{ mol/kg} \approx 4.4 \text{ mol/kg}$

Question: The value of K becomes 5 times when temperature changes from 27° C to 53° C. Calculate E_a .

Options:

(a) 49.41 kJ

(b) 50.3 kJ

(c) 48 kJ

(d) 40 kJ

Answer: (a) Solution:

$$K_{1} = Ae^{-\frac{E_{a}}{RT}}$$
$$\frac{K_{2}}{K_{1}} = \frac{e^{-\frac{E_{a}}{RT_{2}}}}{e^{-\frac{E_{a}}{RT_{1}}}}$$
$$\frac{K_{2}}{K_{1}} = e^{\frac{E_{a}}{R}} \left[\frac{1}{T_{2}} - \frac{1}{T_{1}}\right]$$
$$5 = e^{\frac{E_{a}}{R}} \left[\frac{1}{T_{2}} - \frac{1}{T_{1}}\right]$$



$$\ln 5 = \frac{-E_{a}}{R} \left[\frac{1}{T_{2}} - \frac{1}{T_{1}} \right]$$

$$2.303 \times \log 5 = \frac{-E_{a}}{8.314} \left[\frac{1}{326} - \frac{1}{300} \right]$$

$$1.58 = \frac{-E_{a}}{10^{-3} \times 8.314} \left[\frac{-26}{300 \times 326} \right]$$

$$E_{a} = \frac{1.58 \times 8.314 \times 10^{-3} \times 300 \times 326}{26}$$

$$= \frac{1.58 \times 8.314 \times 0.3 \times 326}{26} = 49.41 \text{ kJ}$$

Question: 1 mole of ideal gas $V_2 = 2V_1$ $PV^{1/2} = constant$ Find T_2/T_1



(a) 1.44

(b) 1.73

(c) 2

(d) 2.23

Answer: (a) Solution:



$$PV^{\frac{1}{2}} = \text{constant}$$

$$PV^{\gamma} = \text{constant} \quad \gamma = \frac{1}{2}$$

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

$$\left(\frac{V_1}{V_2}\right)^{-0.5} = \frac{T_2}{T_1}$$

$$\sqrt{\frac{V_2}{V_1}} = \frac{T_2}{T_1}$$

$$\sqrt{\frac{2V_1}{V_1}} = \frac{T_2}{T_1}$$

$$\therefore \frac{T_2}{T_1} = 1.44$$

Question: Carbylamine product of







Question: Which of the following is false about hydrophilic sols? **Options:**

(a) These cannot be coagulated easily

- (b) They are irreversible in nature
- (c) Their viscosity is similar to that of water

(d) They need electrolytes for stability

Answer: (b)

Solution: Hydrophilic sols are reversible in nature

Question: Statement 1: pH of rain is 5.6

Statement 2: If pH of rain water is less than 5.6, it is called acid rain.
Options:

(a) Both statements are true
(b) S1 is true, S2 is false
(c) S1 is false, S2 is false
(d) Both false

Answer: (a)
Solution: pH of rain water is 5.6

Question: Statement 1: DMG is used to detect Ni²⁺
Statement 2: DMG is a bidentate neutral ligand
Options:

(a) Both statements are true
(b) S₁ is true, S₂ is false
(c) S₁ is false, S₂ is false
(d) Both false

Answer: (b)
Solution: DMG is used to detect Ni⁺² and DMG is bidentate also, but it is anionic ligand

Question: Which molecule does not have same bond length?



Options:

(a) $BF_{4^{-}}$ (b) SF_{4} (c) NH_{3} (d) $IF_{4^{-}}$ Answer: (b) Solution: $SF_{4} \Rightarrow$ It has sp³d hybridization \therefore It has different bond length

Question: BaCO₃, CaCO₃, SrCO₃, MgCO₃ arrange these salts according to their decreasing thermal stability?

Options:

(a) $MgCO_3 > CaCO_3 > SrCO_3 > BaCO_3$ (b) $MgCO_3 > SrCO_3 > CaCO_3 > BaCO_3$ (c) $BaCO_3 > SrCO_3 > CaCO_3 > MgCO_3$ (d) $BaCO_3 > SrCO_3 > MgCO_3 > CaCO_3$ Answer: (c)

Solution: On moving down the group, the electropositive character of alkaline thermal stability also increases

Question: When ethylene glycol is heated with oxalic acid in the presence of conc. H₂SO₄ the product formed is

Options:



Answer: (a) Solution:





Question:



Product is? **Options:**





Answer: (c) Solution:





Question: Compound added for detection of halogen before adding AgNO3

Options:

(a) HNO₃

(b) H_2SO_4

(c) HCl

(d) Can't be determined

Answer: (a)

Solution: HNO₃ is added before AgNO₃ to neutralize the excess NaOH present during the preparation of sodium fusion extract

Question: Order of Magnetic moment of the following: $[FeF_6]^{3-}$, $[Co(NH_3)_6]^{3+}$, $[NiCl_4]^{2-}$, $[Cu(NH_3)_6]^{2+}$ Options: (a) $[FeF_6]^{3-} > [NiCl_4]^{2-} > [Cu(NH_3)_6]^{2+} > [Co(NH_3)_6]^{3+}$ (b) $[FeF_6]^{3-} < [NiCl_4]^{2-} < [Cu(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+}$ (c) $[FeF_6]^{3-} > [Cu(NH_3)_6]^{2+} > [NiCl_4]^{2-} > [Co(NH_3)_6]^{3+}$ (d) $[Co(NH_3)_6]^{3+} > [FeF_6]^{3-} > [NiCl_4]^{2-} > [Cu(NH_3)_6]^{2+}$ Answer: (a)

Solution:

Compounds	No. of unp <mark>aired e</mark> lectrons	
$[FeF_{6}]^{3-}$	5	
$[Co(NH_3)_6]^{3+}$	0	
[NiCl4] ²⁻	2	
$[Cu(NH_3)_6]^{2+}$	1	



JEE-Main-25-02-2021-Shift-2 (Memory Based) MATHEMATICS

Question: cosec $2\cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)$ equals **Options:** (a) (b) (c) (d) Answer: () Solution: $\operatorname{cosec}\left| 2 \operatorname{cot}^{-1}(5) + \operatorname{cot}^{-1}\left(\frac{4}{5}\right) \right|$ $= \operatorname{cosec} \left[2 \tan^{-1} \left(\frac{1}{5} \right) + \tan^{-1} \left(\frac{3}{4} \right) \right]$ $\therefore 2 \tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ $= \csc \left[\tan^{-1} \left(\frac{\frac{2}{5}}{1 - \frac{1}{25}} \right) + \tan^{-1} \left(\frac{3}{4} \right) \right]$ $= \operatorname{cosec}\left[\tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{3}{9}\right) \right]$ $\therefore \tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$ $= \operatorname{cosec} \left| \tan^{-1} \left(\frac{\frac{5}{12} + \frac{3}{4}}{1 - \frac{15}{49}} \right) \right|$ $= \operatorname{cosec} \left| \tan^{-1} \left(\frac{56}{33} \right) \right|$ $= \operatorname{cosec} \left[\operatorname{cosec}^{-1} \left(\frac{65}{56} \right) \right] = \frac{65}{56}$



Question:
$$F(x) = \frac{5^x}{5^x + 5}$$
 find the sum of $f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) \dots f\left(\frac{39}{20}\right)$
Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:
 $f(x) = \frac{5^x}{5^x + 5}, f(2x) = \frac{5^{2-x}}{5^{2-x} + 5} = \frac{5}{5 + 5^x}$
 $\therefore \overline{f(x) + f(2-x) = 1}$
 $\Rightarrow \left[f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right)\right] + \left[f\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right)\right] + \dots + \left[f\left(\frac{19}{20}\right) + f\left(\frac{21}{20}\right)\right] + f\left(\frac{20}{20}\right)$
 $\Rightarrow 19 + \frac{1}{2} = \frac{39}{2}$

Question: If 1-2i is a root of $Z^2 + \alpha z + \beta = 0$ then find $\alpha - \beta$ Options:

(a) (b) (c) (d) Answer: () Solution: $\therefore 1-2i$ is the root of $z^2 + \alpha z + \beta$ $\therefore (1-2i)^2 + \alpha (1-2i) + \beta = 0$ $1-4-4i + \alpha - 2i\alpha + \beta = 0$ $(\alpha + \beta - 3) - i(4 + 2\alpha) = 0$ $\therefore 2\alpha + 4 = 0 \Rightarrow \alpha = -2$ $\alpha + \beta - 3 = 0 \Rightarrow \beta = 5$

Thus,
$$\alpha - \beta = -7$$

Question: $0 < x, y < \pi, \cos x + \cos y - \cos(x + y) = \frac{3}{2}$ find $\sin x + \cos y$

Options:

(a) $\frac{1+\sqrt{3}}{2}$



(b) $\frac{1}{2}$ (c) $\frac{1-\sqrt{3}}{2}$ (d) Answer: (a) Solution: $\cos x + \cos y - \cos \left(x + y \right) = \frac{3}{2}$...(1) $2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - \left[2\cos^2\left(\frac{x+y}{2}\right) - 1\right] = \frac{3}{2}$ $2\cos^2\left(\frac{x+y}{2}\right) - 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) + \frac{1}{2} = 0$: It is quadratic of $\cos\left(\frac{x+y}{2}\right)$ having real roots $\therefore D \ge 0$ $\Rightarrow 4\cos^2\left(\frac{x-y}{2}\right) \ge 4$ $\Rightarrow \cos^2\left(\frac{x-y}{2}\right) \ge 1$ \therefore Max value of $\cos\theta$ is 1 $\therefore \cos^2\left(\frac{x-y}{2}\right) = 1$ $\Rightarrow x = y$ Question: $\lim_{n \to \infty} \frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(n+n-1)^2}$ **Options:** (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) $\frac{1}{3}$ Answer: (a) Solution: $=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{\left(n+r\right)^{2}}$

$$= \lim_{n \to \infty} \sum_{r=0}^{n} \frac{n}{\left(1 + \frac{r}{n}\right)^2} \cdot \frac{1}{n}$$



$$= \int_{0}^{1} \frac{1}{(1+x)^{2}} dx$$
$$= \left[\frac{-1}{1+x}\right]_{0}^{1}$$
$$= \frac{-1}{2} + 1$$
$$= \frac{1}{2}$$

Question: A is a 3×3 matrix and |A| = 4. Operation $R_2 \rightarrow 2R_2 + R_3$ is applies on 2A to get new matrix B. Find |B|

Options:

(a) 16 (b) 64 (c) 80 (d) 128 **Answer: (b) Solution:** $\therefore |A| = 4$

 $\therefore |2A| = 8 \times 4 = 32 \quad {:: 3 \times 3 \text{ matrix}}$

To apply the operation $R_2 \rightarrow \frac{2R_2 + R_3}{R_2 + R_3}$, second row should be divided by 2 to get |B|.

I.e.,
$$\frac{1}{2}|B| = |2A| = 32$$

Thus, $|B| = 64$

Question: Minimum value of : $a^{ax} + a^{1-ax}$; a > 0**Options:**

(a) $\sqrt{a} + 1$ (b) $2\sqrt{a}$ (c) \sqrt{a} (d) a + 1Answer: (b) Solution: $\therefore a > 0$ By applying $AM \ge GM$ $\frac{a^{ax} + a^{1-ax}}{2} \ge \sqrt{a}$ $\Rightarrow a^{ax} + a^{1-ax} \ge 2\sqrt{a}$ Thus, min value is $2\sqrt{a}$



Question:
$$I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x \, dx$$
. Find relation between $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$

Options:

(a) AP
(b) GP
(c) Reciprocal are in AP
(d)
Answer: (c)
Solution:

$$I_{n} + I_{n+2} = \int_{\pi/4}^{\pi/2} \cot^{n} x \, dx + \int_{\pi/4}^{\pi/2} \cot^{n+2} x \, dx$$
$$= \int_{\pi/4}^{\pi/2} \cot^{n} x \left(1 + \cot^{2} x\right) dx$$
$$= \int_{\pi/4}^{\pi/2} \cot^{n} x \, \csc^{2} x \, dx$$

Let $\cot x = t \Longrightarrow -\csc^2 x \, dx = dt$

$$\Rightarrow I_n + I_{n+2} = -\int_1^0 t^n dt$$

$$= \left[\frac{t^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$$

This is general term of H.P.

Hence, $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ are in H.P.

Question: What will be contrapositive of statement, "If we work hard, then we will earn money".

Options:

(a) If you do not earn money then you don't work hard

(b) If you do not earn money then you work hard

(c) If you earn money then you work hard

(d) If you earn money then you don't work hard

Answer: (a)

Solution:

Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

Hence, contrapositive of given statement is "If we will not earn money, then we do not work hard".



Question: A number is selected from 4 digit natural numbers containing exactly one digit as '7'. Find the probability that number when divided by '5' leaves remainder '2'. **Options:**

- (a)
- (b)
- (c) (d)
- (u)

Answer: () Solution:

Total number of 4-digit natural number containing exactly one 7 is

- (a) When 7 is at first place = $1 \times 9 \times 9 \times 9 = 729$
- (b) When 7 is not at first place $= 8 \times 3 \times 9 \times 9 = 1944$
- :. Total number of cases = 2673 (729 + 1944)

Now, A number leaves remainder '2' when divided by '5' only when its unit digit is '2' or '7'

Case-I: when unit digit is 7

Number of favourable cases $= 1 \times 8 \times 9 \times 9 = 648$

Case-II: when unit digit is 2

(a) when first digit is $7 \Rightarrow 1 \times 19 \times 9 = 81$

(b) when first digit is not $7 \Rightarrow 1 \times 8 \times 2 \times 9 = 144$

 \therefore Total number of favourable cases = 648 + 81 + 144 = 873

 \therefore Required probability = $\frac{\text{favourable case}}{1}$

	Total case	
_	873	
_	2673	
_	97	
_	279	

Question: Set A contain 3 elements, set B contain 5 elements, number of one-one functions from A to B is 'x' and number of one-one functions from A to $A \times B$ is 'y' then **Options:**

(a) (b) (c) (d) Answer: () Solution:

$x = {}^{5}C_{3} \times 3!$

 $y = {}^{15}C_3 \times 3!$

(:: $A \times B$ has 15 elements)

$$\Rightarrow \frac{y}{x} = \frac{15 \times 14 \times 13}{5 \times 4 \times 3} = \frac{7 \times 13}{2}$$
$$\Rightarrow 2y = 91x$$



Question: The shortest distance between the line x - y = 1 and the curve $x^2 = 2y$ **Options:**

(a) (b) (c) (d) Answer: () Solution:(i) $x^2 = 2y$ Differentiating w.r.t x $2x = 2\frac{dy}{dx}$ $\frac{dy}{dx} = x$ For shortest distance $\frac{dy}{dx} = x = \text{ slope of line } (x - y = 1)$ $\Rightarrow x = 1$ Put in (i) $y = \frac{1}{2}$ So, shortest distance = distance of point $\left(1,\frac{1}{2}\right)$ from line $\left(x-y=1\right)$ $= \frac{1 - \frac{1}{2} - 1}{\sqrt{2}}$ $=\frac{1}{2\sqrt{2}}$

Question: Let α and β be the roots of $x^2 - 6x - 2 = 0$ with $\alpha > \beta$ if $a_n = \alpha^n - \beta^n$ for $n \ge 1$ then the value of $\frac{a_{10} - 2a_8}{3a_9}$ **Answer: 2.00 Solution:**

 $\therefore \alpha \& \beta \text{ are roots of } x^2 - 6x - 2 = 0$ $\therefore \alpha^2 = 6\alpha + 2 \& \beta^2 = 6\beta + 2$ Now, $\frac{a_{10} - 2a_8}{3a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)}$



$$=\frac{\alpha^{8}\left(\alpha^{2}-2\right)-\beta^{8}\left(\beta^{2}-2\right)}{3\left(\alpha^{9}-\beta^{9}\right)}=\frac{6\left(\alpha^{9}-\beta^{9}\right)}{3\left(\alpha^{9}-\beta^{9}\right)}=2$$

Question: $\lim_{x\to 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)} = b$ find a - 2b?

Answer: 5.00 Solution:

$$\lim_{x \to 0} \frac{ax - e^{4x} + 1}{ax \left[\frac{e^{4x} - 1}{4x}\right] \times 4x} \qquad \left\{ \because \lim_{h \to 0} \frac{e^h - 1}{h} = 1 \right\}$$

$$\Rightarrow \lim_{x \to 0} \frac{ax - e^{4x} + 1}{4ax^2}$$

Applying L-Hospital's Rule,

$$\Rightarrow \lim_{x \to 0} \frac{a - 4e^{4x}}{8ax} \Rightarrow a = 4 \text{ as } \left(\frac{0}{0}\right) \text{ form}$$

Apply L-Hospital's Rule

$$\Rightarrow \lim_{x \to 0} \frac{-16e^{4x}}{8a} = \frac{-1}{2} = b$$

Thus, a - 2b = 4 + 1 = 5

Question: If $x = y^4$ and xy = k, cut each other at right angle then find $(4k)^6 =$

Answer: 4.00 Solution:

 $\therefore x = y^4$ & xy = k cut each other perpendicularly

$$y = x^{1/4}$$

$$y = \frac{k}{x}$$

$$m_1 = \frac{dy}{dx} = \frac{1}{4x^{3/4}}$$

$$m_2 = \frac{dy}{dx} = \frac{-k}{x^2}$$

$$\therefore m_1 \cdot m_2 = -1 \Rightarrow \frac{-k}{4x^{\frac{11}{4}}} = -1$$

$$\Rightarrow k = 4x^{\frac{11}{4}}$$
or
$$x = \left(\frac{k}{4}\right)^{\frac{4}{11}}$$

$$y = (x)^{\frac{1}{4}} = \left(\frac{k}{4}\right)^{\frac{1}{11}}$$



Thus, xy = k $\Rightarrow \left(\frac{k}{4}\right)^{\frac{5}{11}} = k$ $\Rightarrow \frac{k^5}{4^5} = k^{11}$ $\Rightarrow k^6 = \frac{1}{4^5}$ $\Rightarrow \left(4k\right)^6 = \frac{4^6}{4^5} = 4$ **Question:** $\int_{-\infty}^{\infty} |3x^2 - 3x - 6| dx$ **Answer: 19.00** Solution: $I = 3\int_{-\infty}^{\infty} |x^2 - x - 2| dx = 3\int_{-\infty}^{\infty} |(x - 2)(x + 1)| dx$ Now, $(x-2)(x+1) > 0 \Rightarrow x \in (-\infty, -1) \cup (2, \infty)$ $\therefore I = 3 \left[\int_{-2}^{-1} (x^2 - x - 2) dx - \int_{-2}^{2} (x^2 - x - 2) dx \right]$ $=3\left[\left(\frac{x^{3}}{3}-\frac{x^{2}}{2}-2x\right)_{-2}^{-1}-\left(\frac{x^{3}}{3}-\frac{x^{2}}{2}-2x\right)_{-1}^{2}\right]$ $=3\left[\left(-\frac{1}{3}-\frac{1}{2}+2\right)-\left(-\frac{8}{3}-2+4\right)-\left(\frac{8}{3}-2-4\right)+\left(-\frac{1}{3}-\frac{1}{2}+2\right)\right]$ $=3\left[-\frac{2}{3}+7\right]=19$

Question: How many 2 digit natural numbers 'x' are there which satisfy $3^x + 7^x$ is divisible by '10'.

Answer: 45.00 Solution: $7^{x} + 3^{x} = (5+2)^{x} + (5-2)^{x}$ $= 2 \Big[{}^{x}C_{0}5^{x} + {}^{x}C_{2}5^{x-2}.2^{2} + {}^{x}C_{4}5^{x-4}.2^{4} + ... \Big]$ Here, if *x* is odd, them last term will be ${}^{x}C_{x-1}5^{1}.2^{x-1}$ So, the number will be divisible by 10 But if *x* is even, then last term be 2^{x}



So, the number will not be divisible by 10

Hence, for all the 2 digit odd natural numbers, $3^x + 7^x$ will be divisible by 10 and number of 2 digit odd natural numbers is 45.

